Cryptographic design: Trapdoor and MPCitH digital signatures

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PQSCA summer school
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TU/e

Trapdoor-based digital signature schemes

Multivariate signatures

Fiat-Shamir construction

trapdoor construction

Examples.

MQDSS

SOFIA

Examples.

HFEv-

UOV

The MQ problem

A quadratic system of m equations in n variables over a finite field $\mathbb{F}_q$:

$$f^{(k)}(x_1, \ldots, x_n) = \sum_{1 \le i \le j \le n} \gamma_{ij}^{(k)} x_i x_j + \sum_{1 \le i \le n} \beta_i^{(k)} x_i + \alpha^{(k)}$$

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→ Hard in general (should be hard for randomly generated instances).

→ Can become easy if we have some structure (a trapdoor).

The trapdoor construction

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- Central map:
  $f : (x_1, \dots, x_n) \in \mathbb{F}_q^n \to \left(f^{(1)}(x_1, \dots, x_n), \dots, f^{(m)}(x_1, \dots, x_n)\right) \in \mathbb{F}_q^m$

- Two bijective linear (or affine) transformations:
  $\mathbf{S} \in \mathrm{GL}_n(\mathbb{F}_q)$ and $\mathbf{T} \in \mathrm{GL}_m(\mathbb{F}_q)$

- Public map:
  $p = \mathbf{T} \circ f \circ \mathbf{S}$

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Main idea:

- The central map has a structure such that it is easy to find preimages: it is easy (polynomial time) to compute $f^{-1}(\mathbf{x})$ for a target vector $\mathbf{x}$.

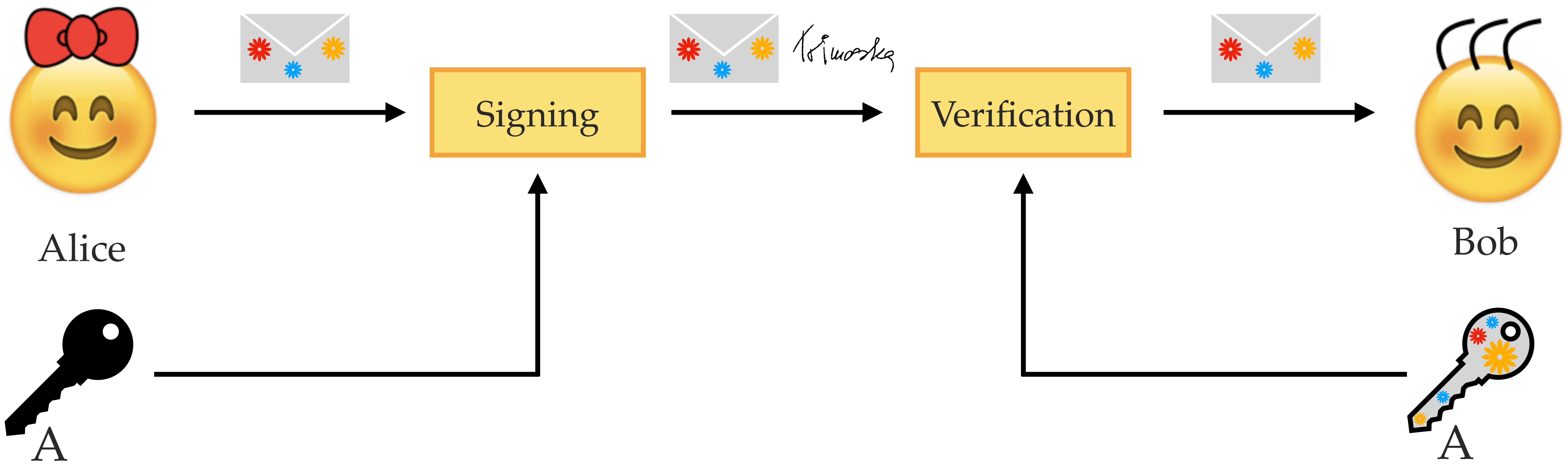
- The linear transformations hide the structure of the central map.

The trapdoor construction

$$
\begin{tikzcd}
\mathbf{w} \in \mathbb{F}_q^m \arrow[r, "\mathbf{T}^{-1}"] & \mathbf{x} \in \mathbb{F}_q^m \arrow[r, "f^{-1}"] & \mathbf{y} \in \mathbb{F}_q^n \arrow[r, "\mathbf{S}^{-1}"] & \mathbf{z} \in \mathbb{F}_q^n \arrow[lll, bend left, "p"]
\end{tikzcd}
$$

General workflow

The trapdoor construction



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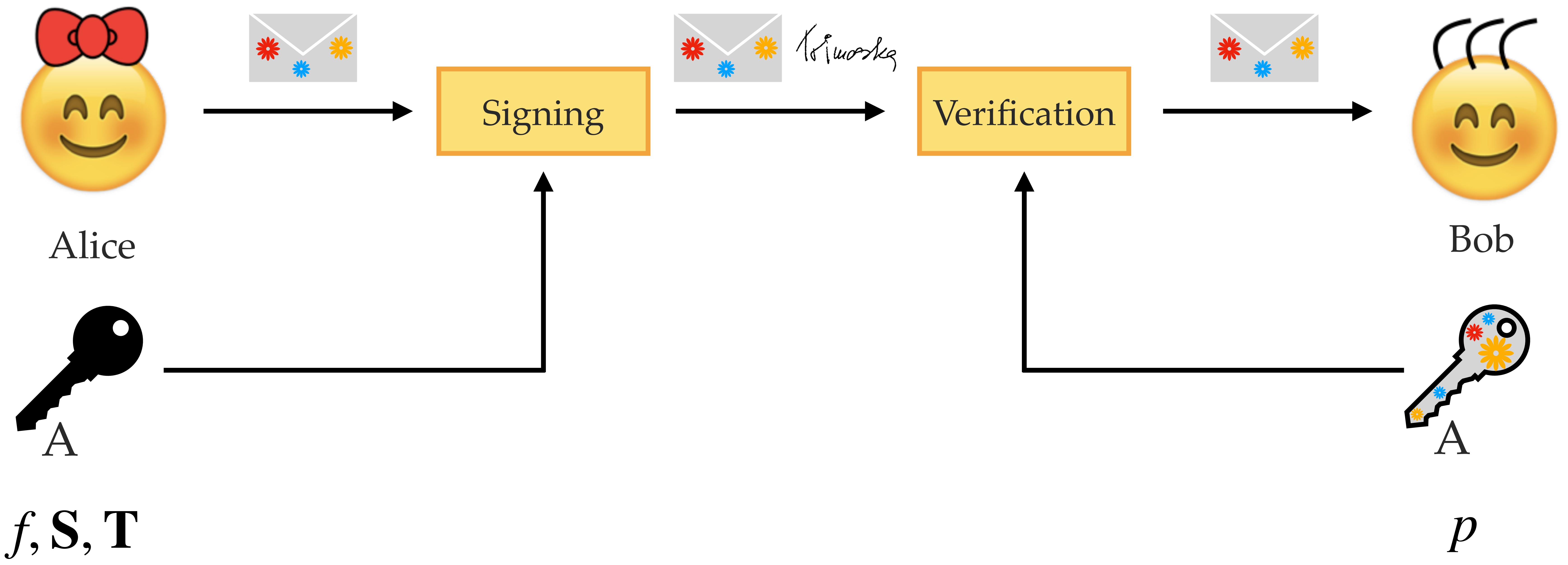

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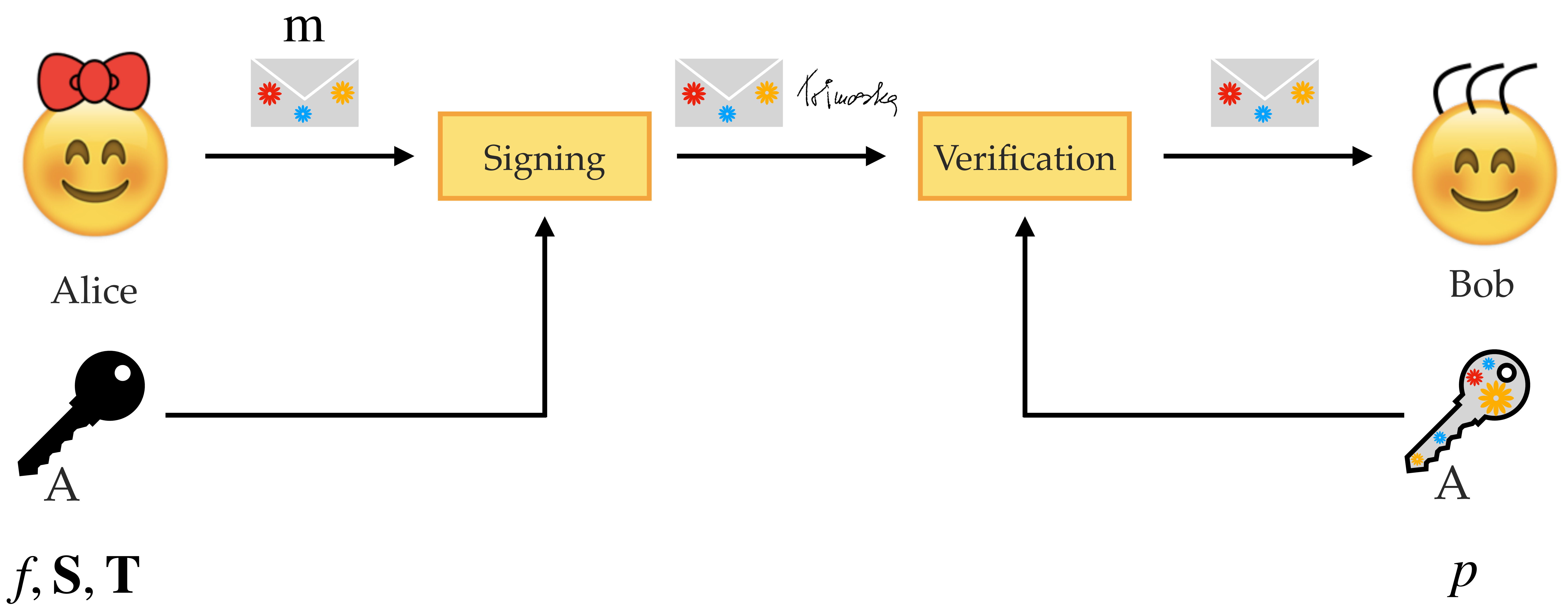

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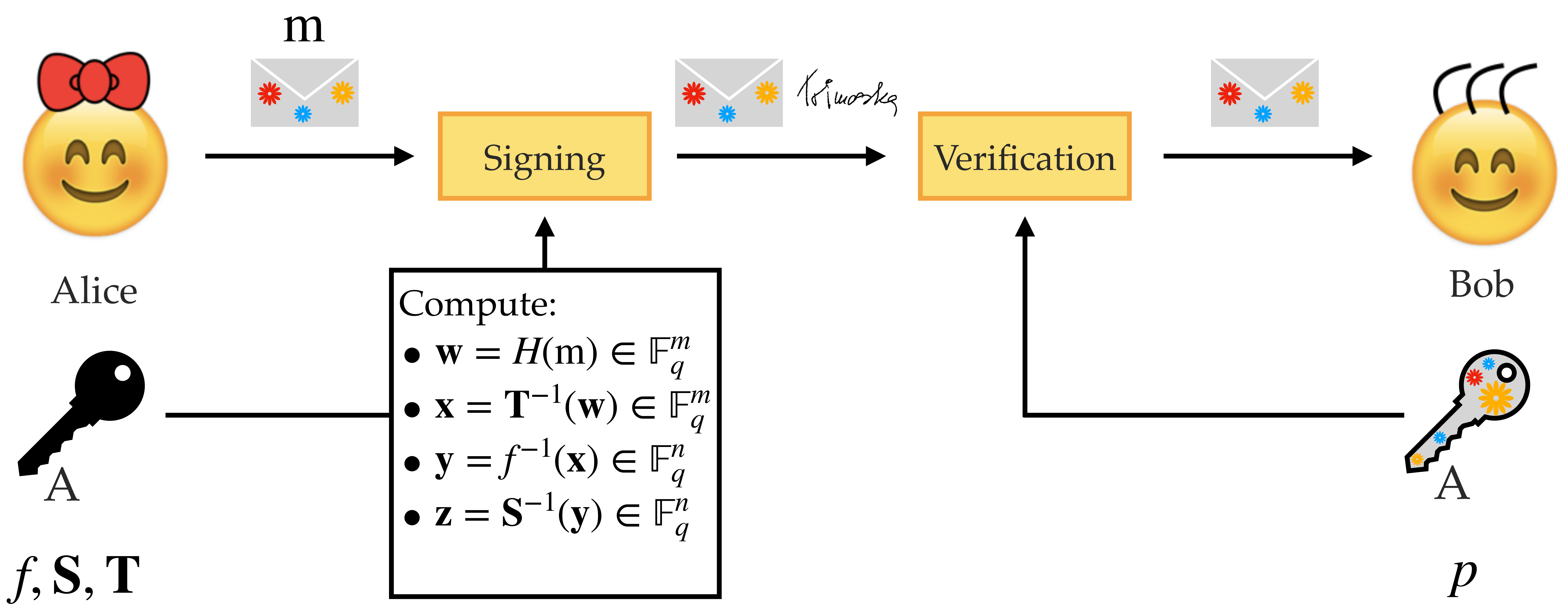

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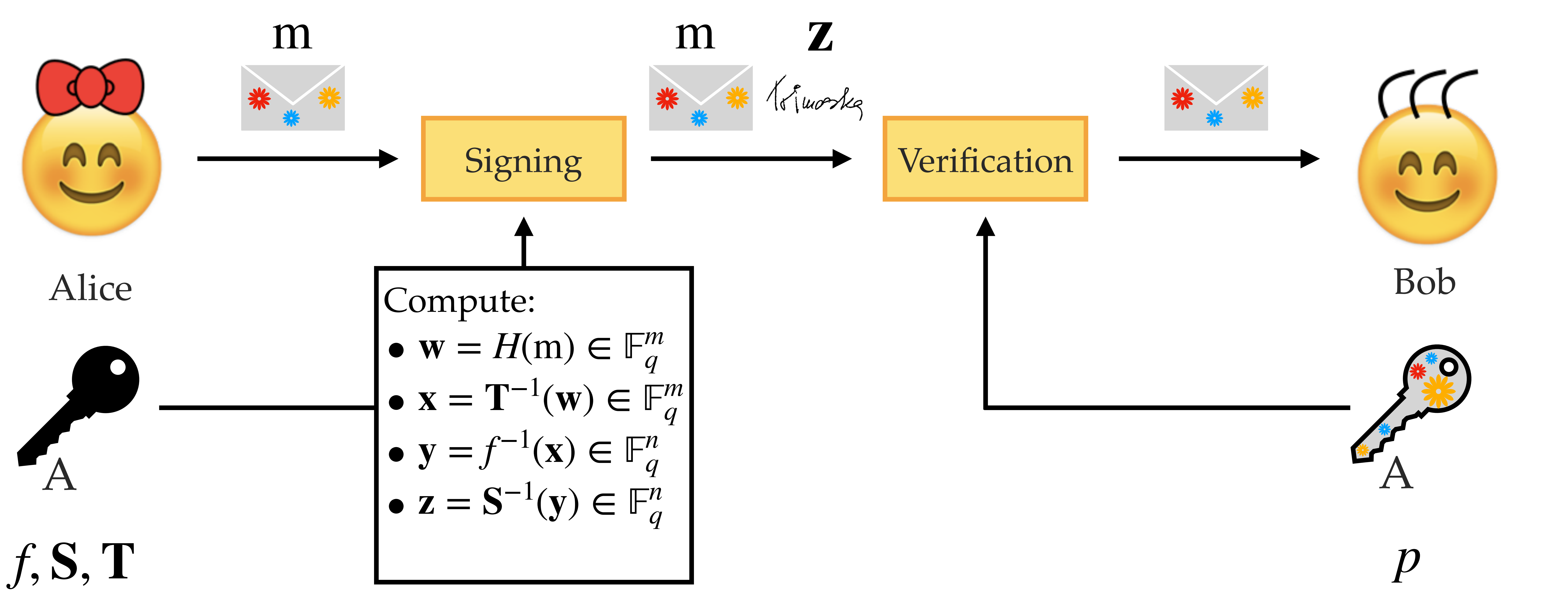

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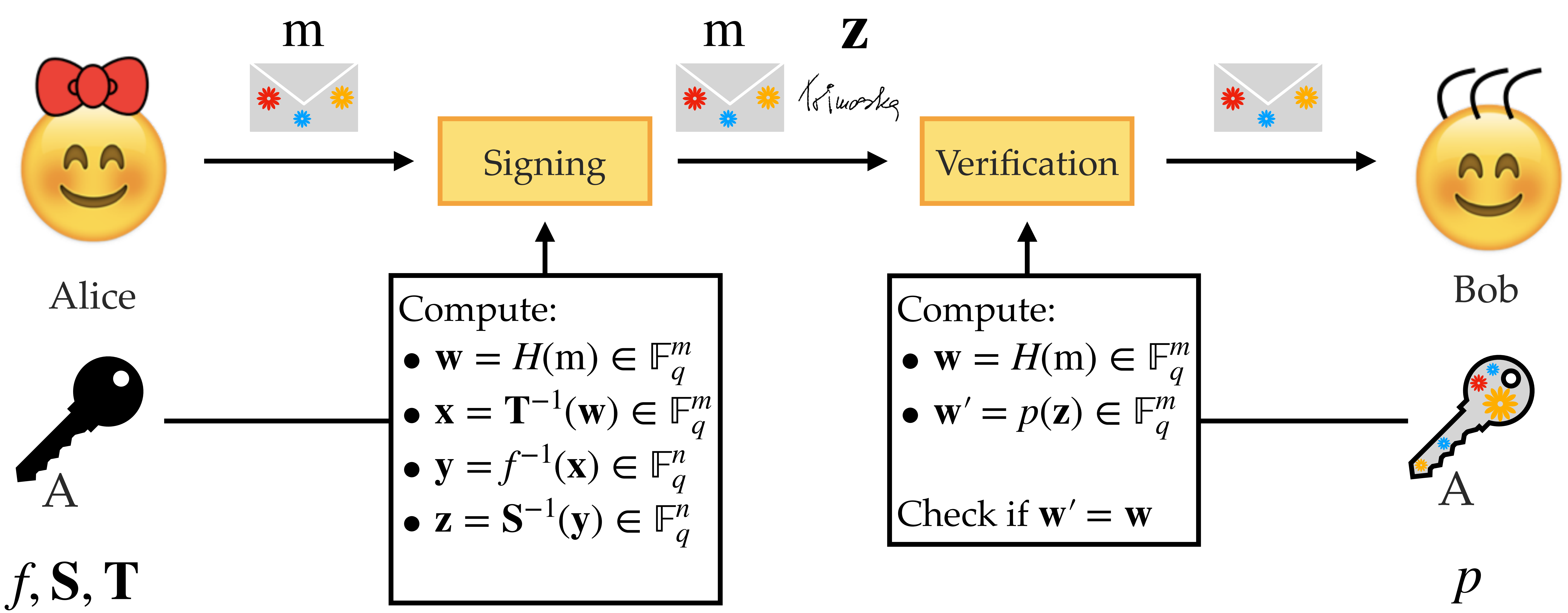

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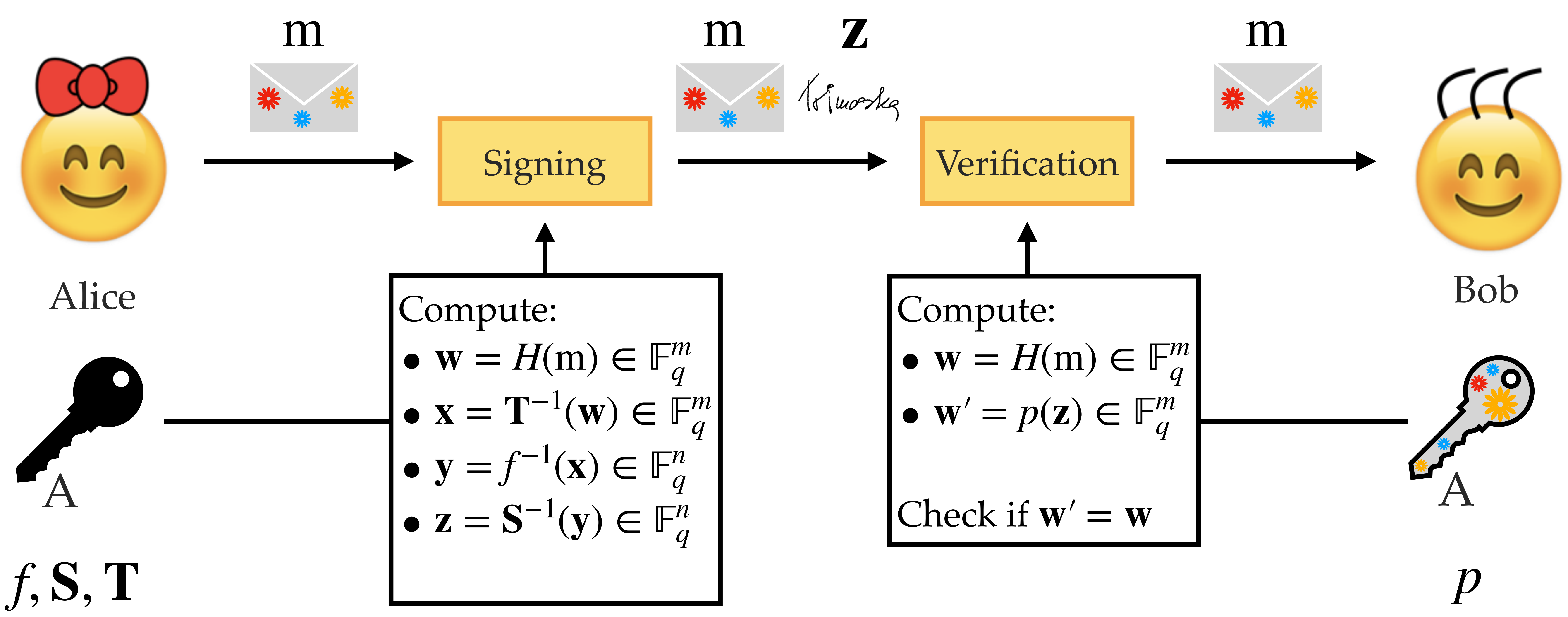

Isomorphism of polynomials

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The Isomorphism of Polynomials (IP) problem

Input: Two m-tuples of multivariate polynomials
$f = (f^{(1)}, \ldots, f^{(m)}), p = (p^{(1)}, \ldots, p^{(m)}) \in \mathbb{F}_q[x_1, \ldots, x_n]^m$.
Question: Find - if any - $\mathbf{S} \in \mathrm{GL}_n(\mathbb{F}_q)$ and $\mathbf{T} \in \mathrm{GL}_m(\mathbb{F}_q)$ such that $p = \mathbf{T} \circ f \circ \mathbf{S}$.

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The Extended Isomorphism of Polynomials (EIP) problem

Input: An m-tuple of multivariate polynomials $p = (p^{(1)}, \ldots, p^{(m)}) \in \mathbb{F}_q[x_1, \ldots, x_n]^m$ and a special class of m-tuples of multivariate polynomials $\mathscr{C} \subseteq \mathbb{F}_q[x_1, \ldots, x_n]^m$.
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Signature schemes with the trapdoor construction rely on EIP, because we do not have the central map f, but we know the special class to which it belongs (example - UOV - coming up).

Unbalanced Oil and Vinegar (UOV)

The UOV central map

Unbalanced Oil and Vinegar [Kipnis, Patarin, Goubin, '99]

$$f^{(k)}(x_1, \ldots, x_n) = \sum_{i \in V, j \in V} \gamma_{ij}^{(k)} x_i x_j + \sum_{i \in V, j \in O} \gamma_{ij}^{(k)} x_i x_j + \sum_{i=1}^{n} \beta_i^{(k)} x_i + \alpha^{(k)}$$

Index set of vinegar variables: $V = \{1, \ldots, v\}$

Index set of oil variables: $O = \{v+1, \ldots, n\}$

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- The central map is constructed in such a way that enumerating all of the vinegar variables leaves us with a linear system in the oil variables (oil does not mix with oil).
- Everything is as described in the previous slides, except that we do not have a linear transformation on the output: $\mathbf{T} = \mathbf{I}$.

Matrix representation of quadratic forms

Quadratic form: $f(\mathbf{x}) = \sum \gamma_{ij} x_i x_j$

$$
\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}}_{\mathbf{x}^\top}
\quad
\underbrace{\begin{bmatrix}
\gamma_{1,1} & \frac{\gamma_{1,2}}{2} & \frac{\gamma_{1,3}}{2} & \frac{\gamma_{1,4}}{2} \\
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\end{bmatrix}}_{\mathbf{F}}
\quad
\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}}
$$

so with $\mathbf{x} = (x_1, \ldots, x_n)$, we get $\mathbf{x}^\top \mathbf{F} \mathbf{x}$.

Matrix representation of bilinear forms

Bilinear form: $f(\mathbf{x}, \mathbf{y}) = \sum \gamma_{ij} x_i y_j$

$$
\overset{\mathbf{x}^\top}{\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}}
\qquad
\overset{\mathbf{B}}{\begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} & \gamma_{1,4} \\ \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} & \gamma_{2,4} \\ \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} & \gamma_{3,4} \\ \gamma_{4,1} & \gamma_{4,2} & \gamma_{4,3} & \gamma_{4,4} \end{bmatrix}}
\qquad
\overset{\mathbf{y}}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}}
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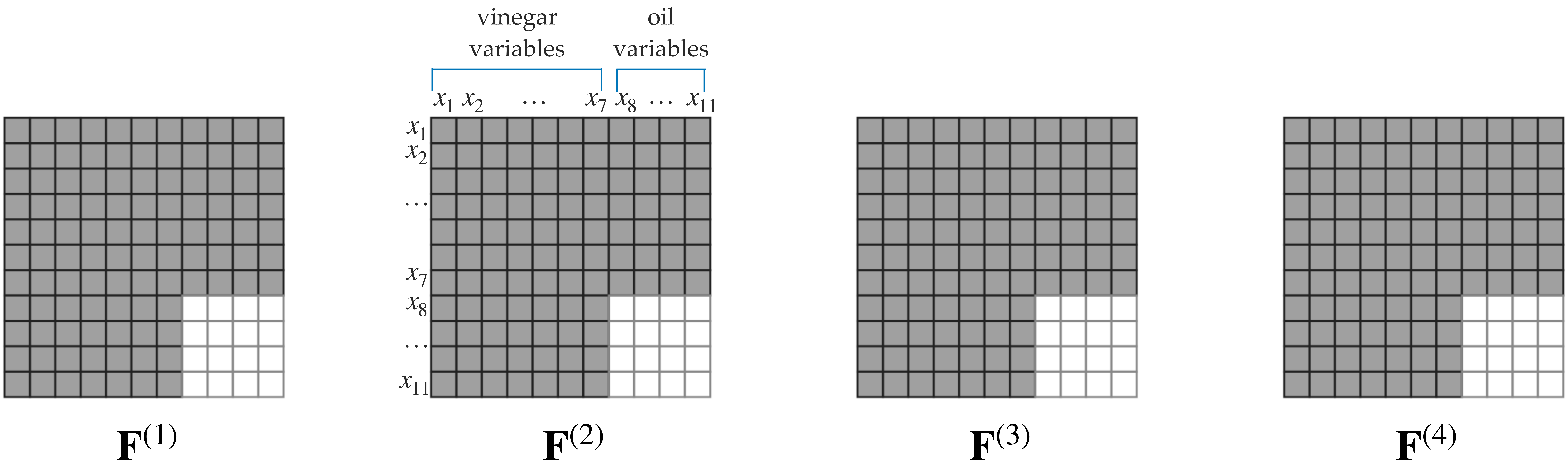
so with $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$, we get $\mathbf{x}^\top \mathbf{B} \mathbf{y}$.

The UOV central map

Toy example: $v = 7, m = 4$



*Grayed areas represent the entries that are possibly nonzero; blank areas denote the zero entries;

UOV key generation

In matrix representation

$$\mathbf{P}^{(k)} = \mathbf{S}^\top \mathbf{F}^{(k)} \mathbf{S}, \text{ for all } k \in \{1, \ldots, m\}.$$

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UOV in the NIST competition

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TUOV
PROV
MAYO
VOX
QR-UOV
SNOVA

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Example.

	NIST S.L.	n	m	q	$\lvert\mathsf{epk}\rvert$ (bytes)	$\lvert\mathsf{esk}\rvert$ (bytes)	$\lvert\mathsf{cpk}\rvert$ (bytes)	$\lvert\mathsf{csk}\rvert$ (bytes)	signatur (bytes)
`uov-Ip`	1	112	44	256	278 432	237 896	43 576	32	128
`uov-Is`	1	160	64	16	412 160	348 704	66 576	32	96
`uov-III`	3	184	72	256	1 225 440	1 044 320	189 232	32	200
`uov-V`	5	244	96	256	2 869 440	2 436 704	446 992	32	260

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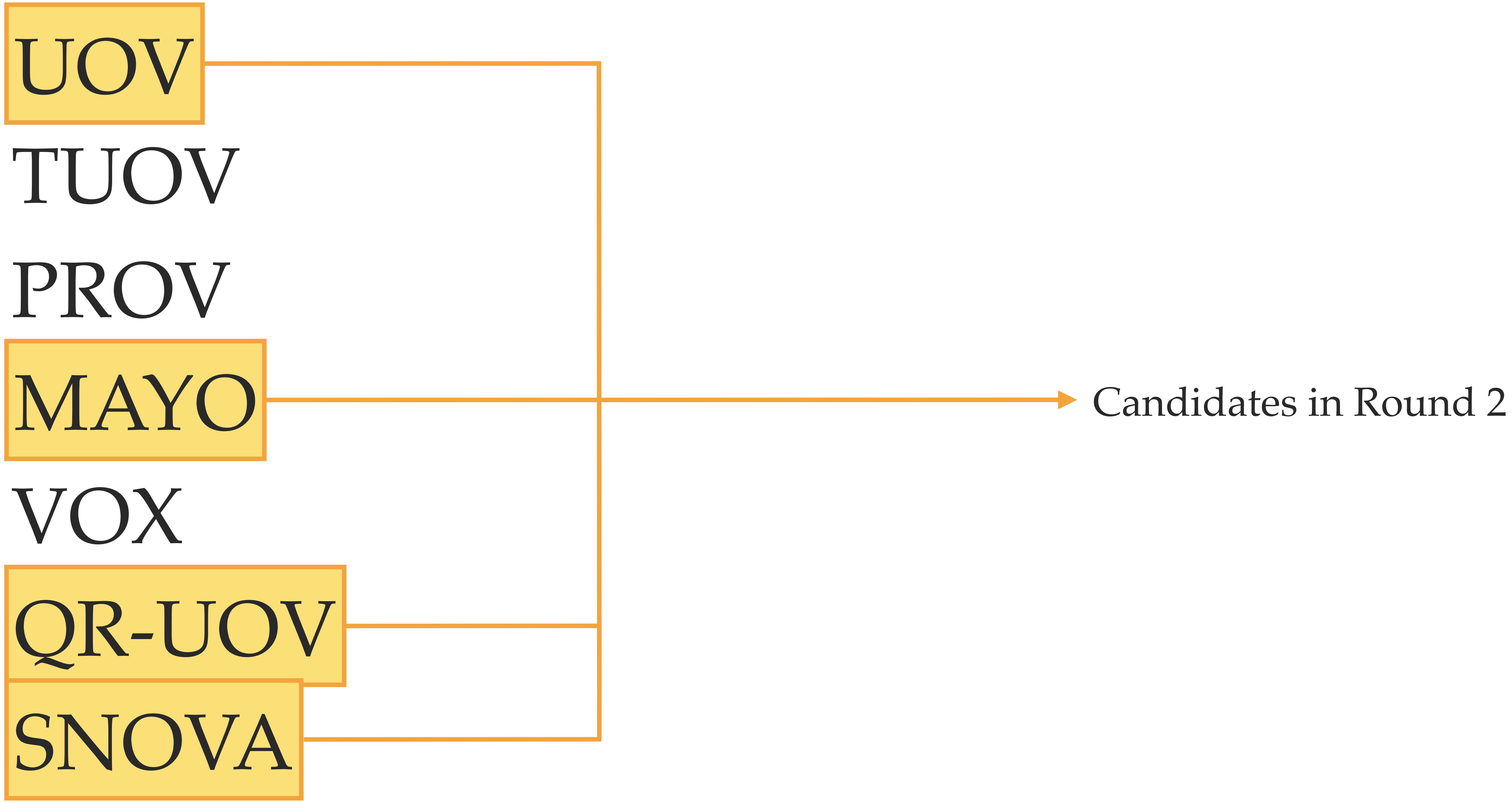
- We choose $n \sim 2.5m$ (slightly bigger than)

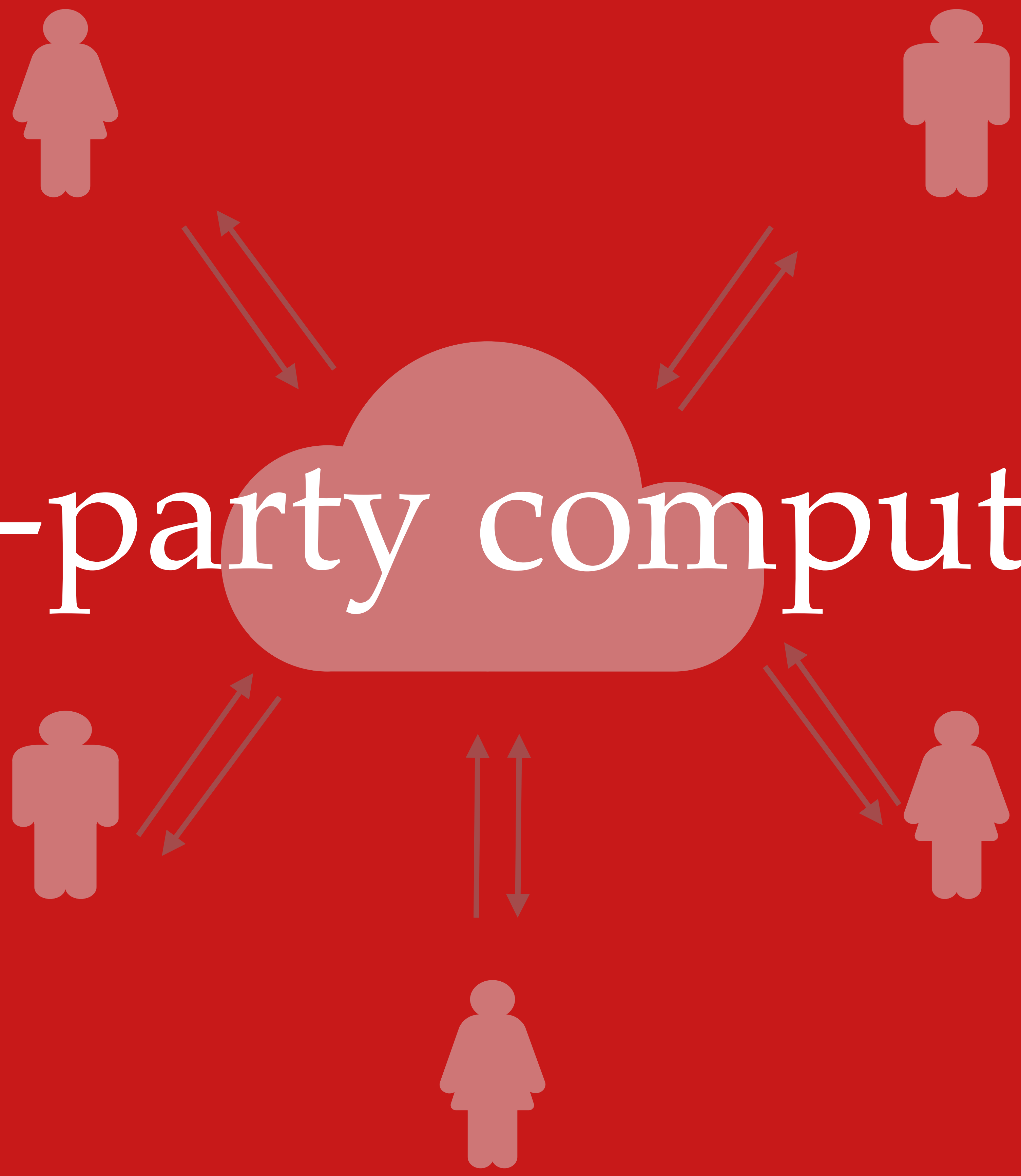
UOV-like schemes have:

- Big public keys
- Small signatures

UOV in the NIST competition



Multi-party computation



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One-way function
$f(x) = y$

MPC protocol

MPCitH
identification
scheme

Digital signature

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Zero-knowledge proof of knowledge: the prover needs to prove that they know a secret x s.t. $f(x) = y$, for a public y, without revealing any information about x.

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→ Standard symmetric primitives

→ MPC-friendly symmetric primitives

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Examples in this talk: Discrete log Syndrome decoding

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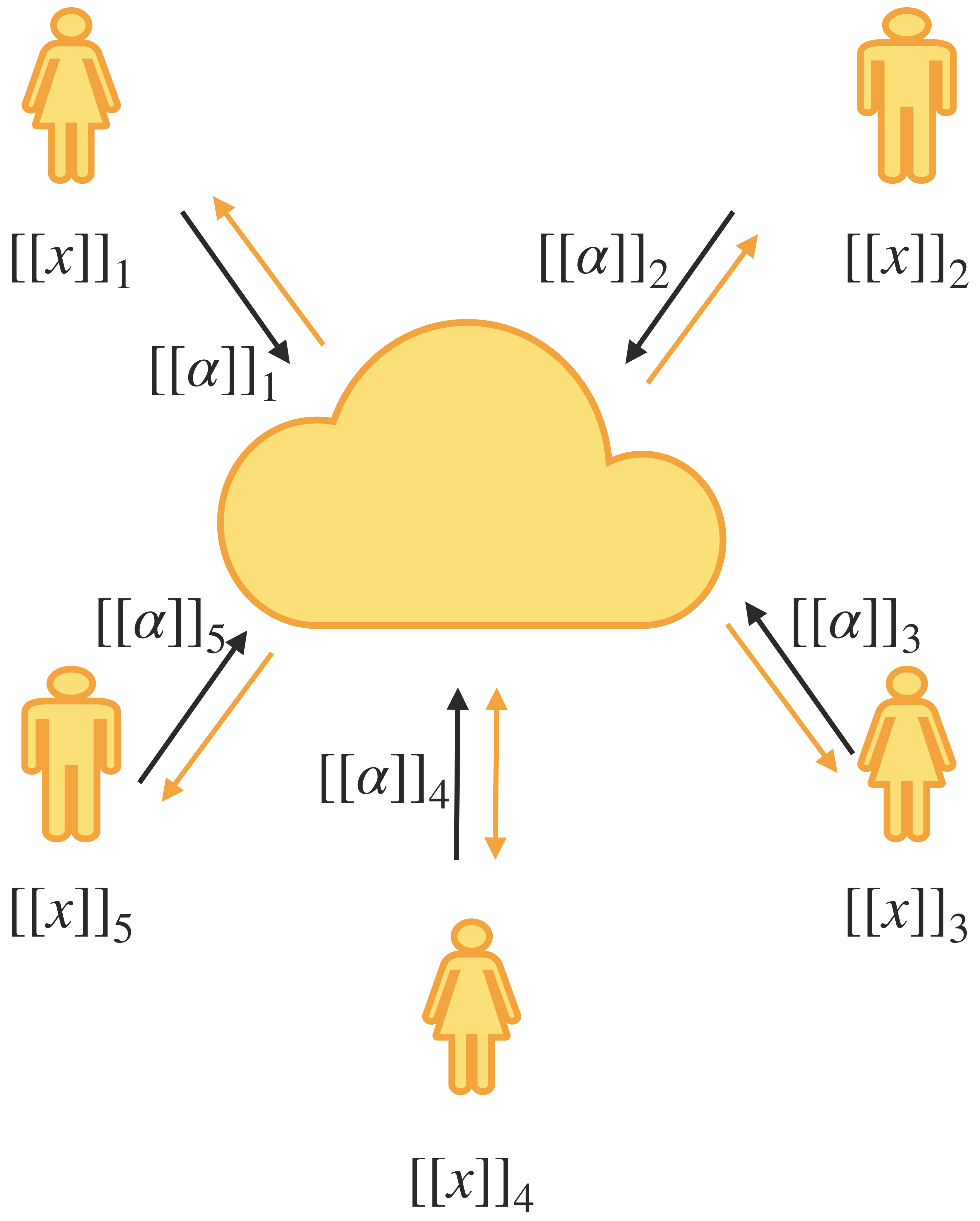
Broadcast model



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Given elements g and h of a finite cyclic group, find x such that $g^x = h$.

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Easy example because it is additively homomorphic.

$$g^{[[x]]_1} \cdot g^{[[x]]_2} \cdot \ldots \cdot g^{[[x]]_N} = g^x,$$

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Jointly check whether $g^x = h$:

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- (After broadcast) check whether $\prod_{i=1}^{N} [[h]]_i = h$.

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! Not post-quantum

Relevant example: syndrome decoding

The syndrome decoding problem

Given $\mathbf{s} \in \mathbb{F}_q^k$ and $\mathbf{H} \in \mathscr{M}_{k\times n}(\mathbb{F}_q)$ and an integer $t \leq n$, find $\mathbf{e} \in \mathbb{F}_q^n$ such that $\mathbf{s} = \mathbf{He}$ and $\mathrm{wt}(\mathbf{e}) = t$.

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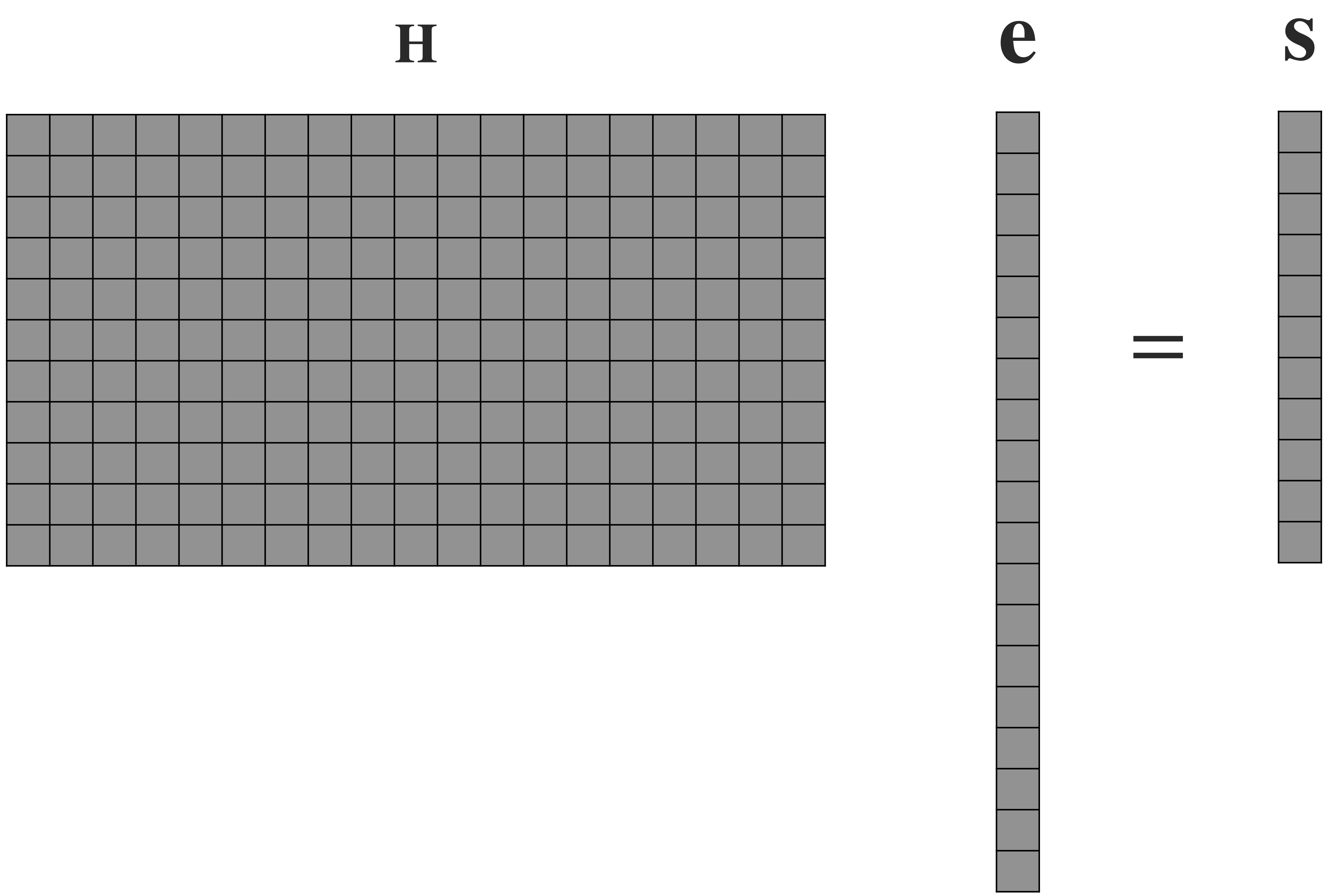

Entry is 0

Entry is 1

Entry is 0 or 1

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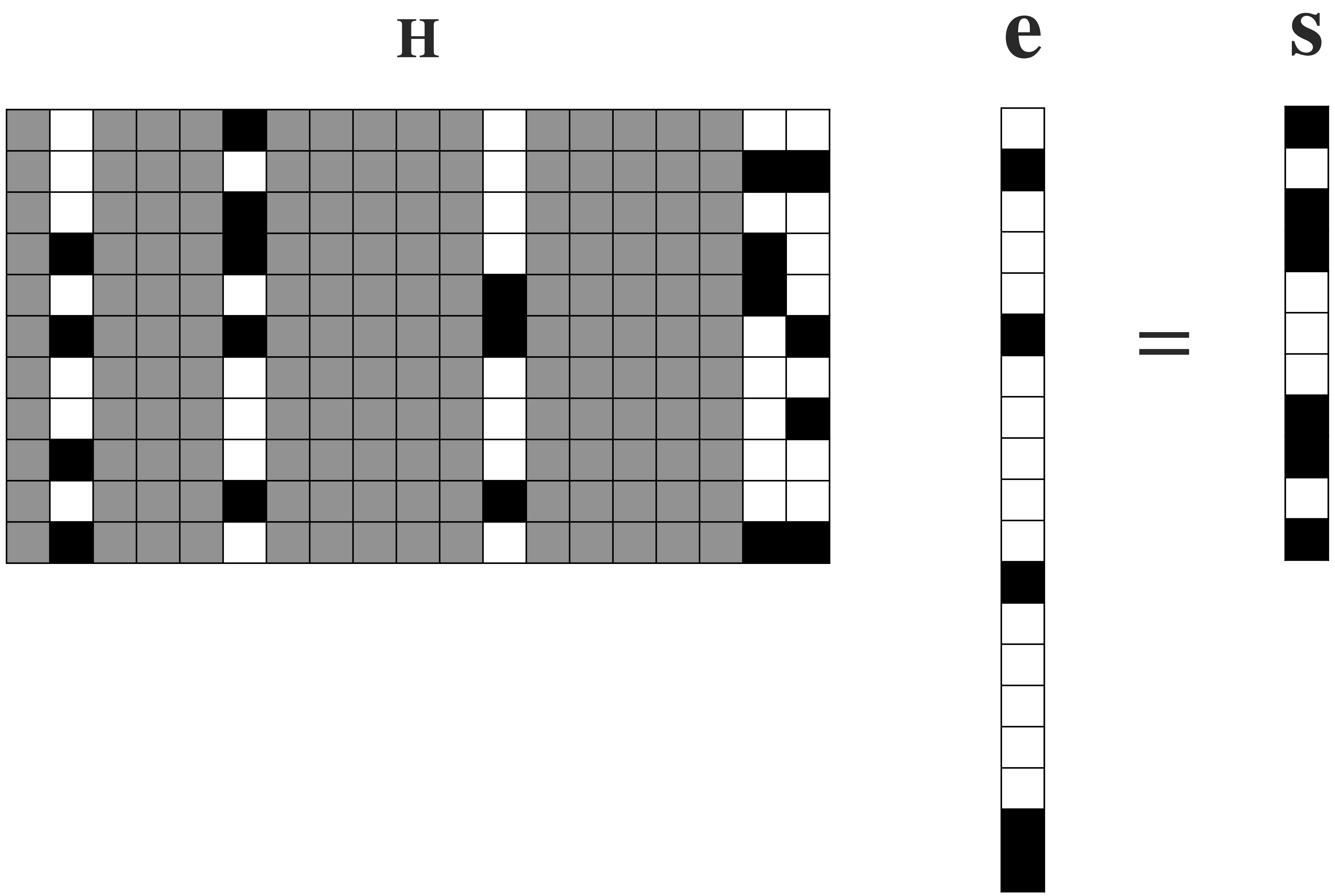

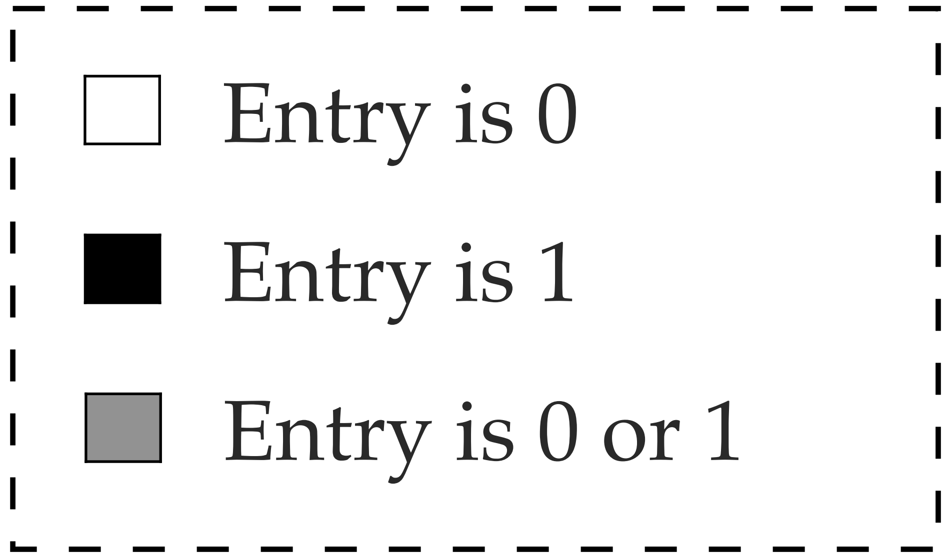

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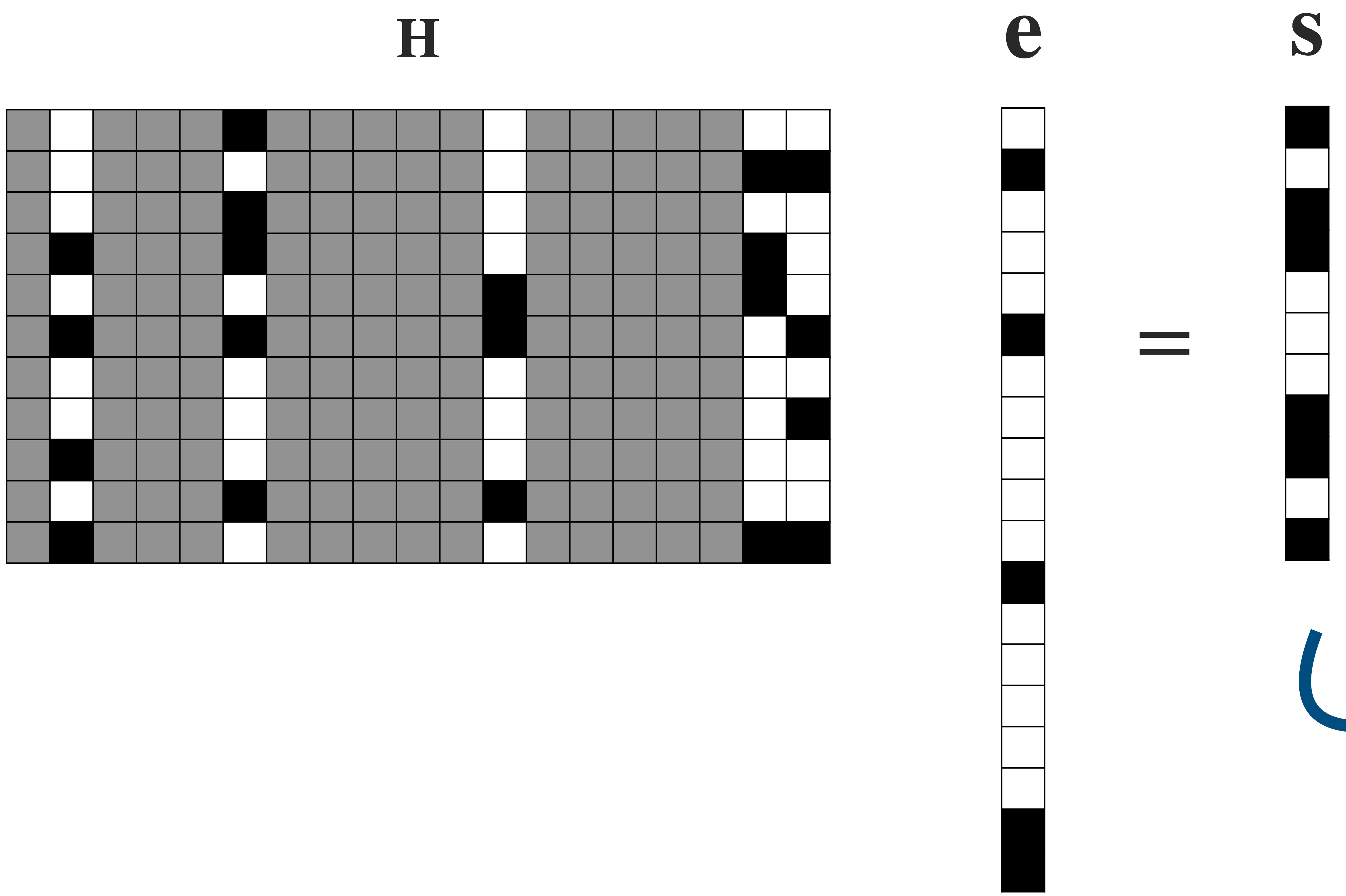
Hamming weight (number of nonzero entries)



Entry is 0

Entry is 1

Entry is 0 or 1

$\mathbf{s}$ is equal to the sum of the columns where e_i is nonzero.

Relevant example: syndrome decoding

The syndrome decoding problem

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Not a 'linear problem' (otherwise, it would be easy).

because of the weight constraint

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Not a 'linear problem' (otherwise, it would be easy).

because of the weight constraint

Finding $\mathbf{e}$ such that $\mathbf{s} = \mathbf{He}$ is easy.

Finding $\mathbf{e}$ such that $\mathrm{wt}(\mathbf{e}) = t$ is easy.

Finding $\mathbf{e}$ that satisfies both constraints is hard.

Relevant example: syndrome decoding

The syndrome decoding problem

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Jointly check whether $\mathbf{s} = \mathbf{He}$:

$\longrightarrow$ Each party computes $[[\mathbf{s}_i]] = \mathbf{H}[[\mathbf{e}_i]]$.

$\longrightarrow$ (After broadcast) check whether $\displaystyle\sum_{i=1}^{N} [[\mathbf{s}_i]] = \mathbf{s}$.

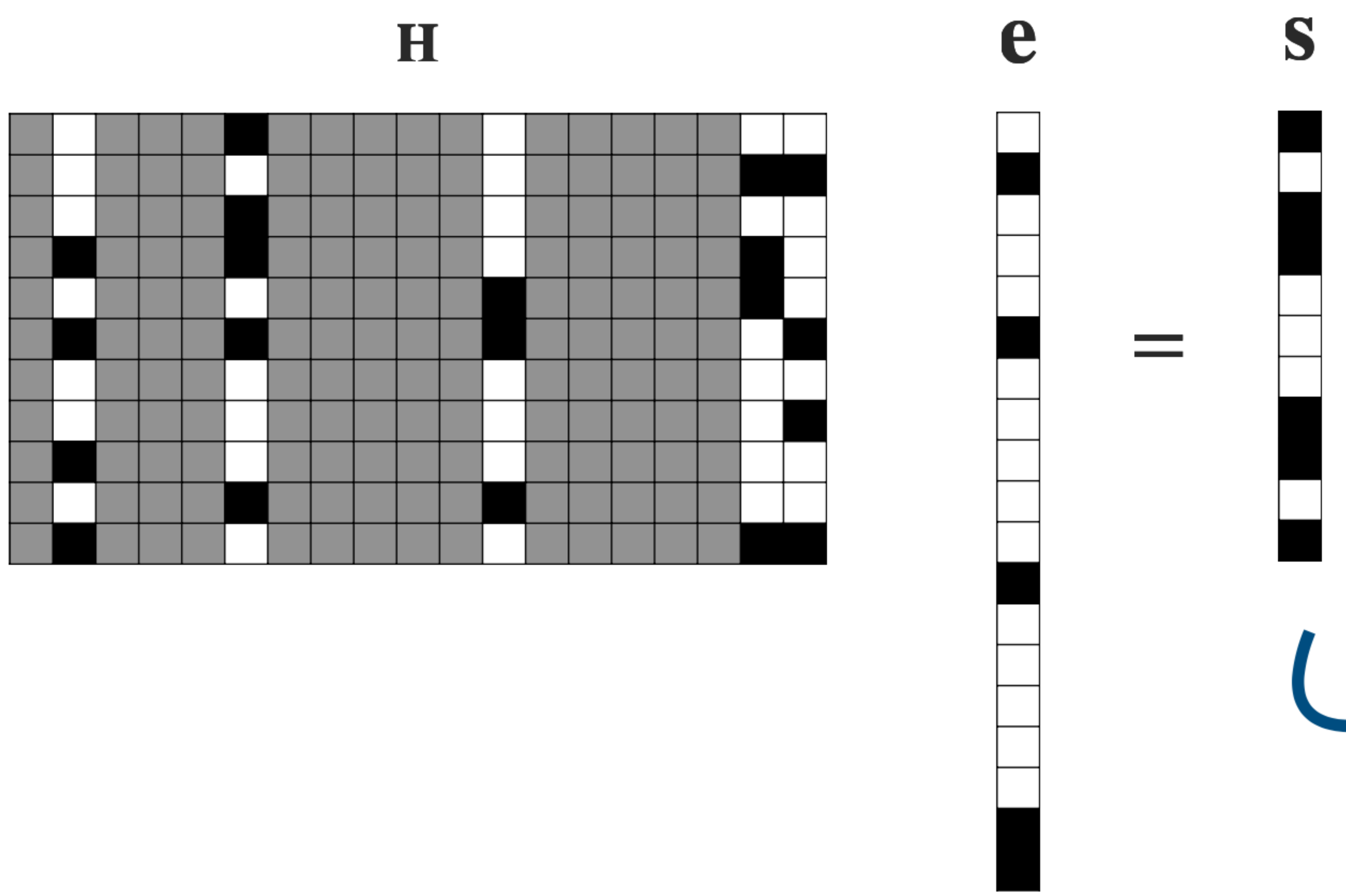
Relevant example: syndrome decoding

The syndrome decoding problem

Jointly check whethe

→ Each pa

→ (After b



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Jointly check whether $\mathrm{wt}(\mathbf{e}) = t$:

→ More complicated.

The MPCitH construction

One-way function
$f(x) = y$

Zero-knowledge proof of knowledge: the prover needs to prove that they know a secret x s.t. $f(x) = y$, for a public y, without revealing any information about x (as in Lecture 3).

MPC protocol

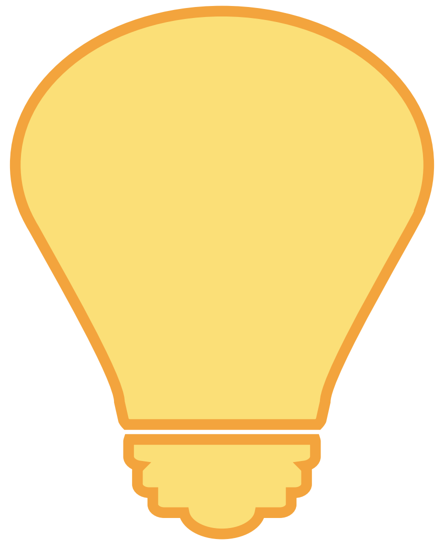
Multi-party computation

MPCitH identification scheme

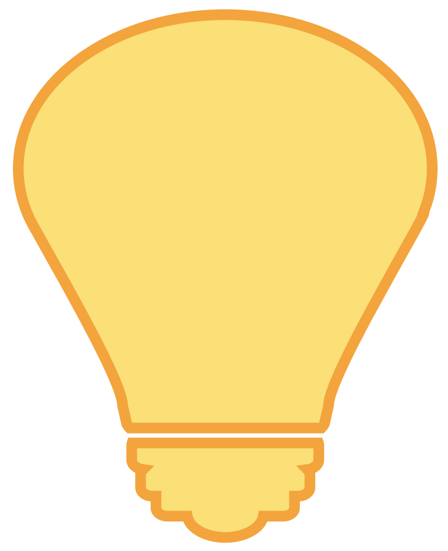
MPC-in-the-Head

Digital signature

MPCitH



MPCitH



→ Properties that the underlying MPC model needs to have:

MPCitH

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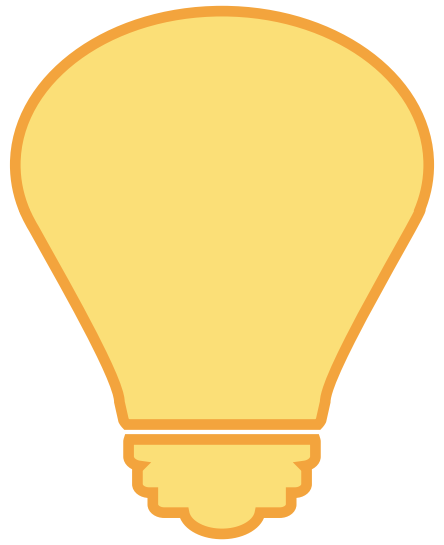
- $(N-1)$-private: the views of any $N-1$ parties do not reveal any information on x.
- Semi-honest: weak notion in MPC, but enough for the MPCitH application.

MPCitH

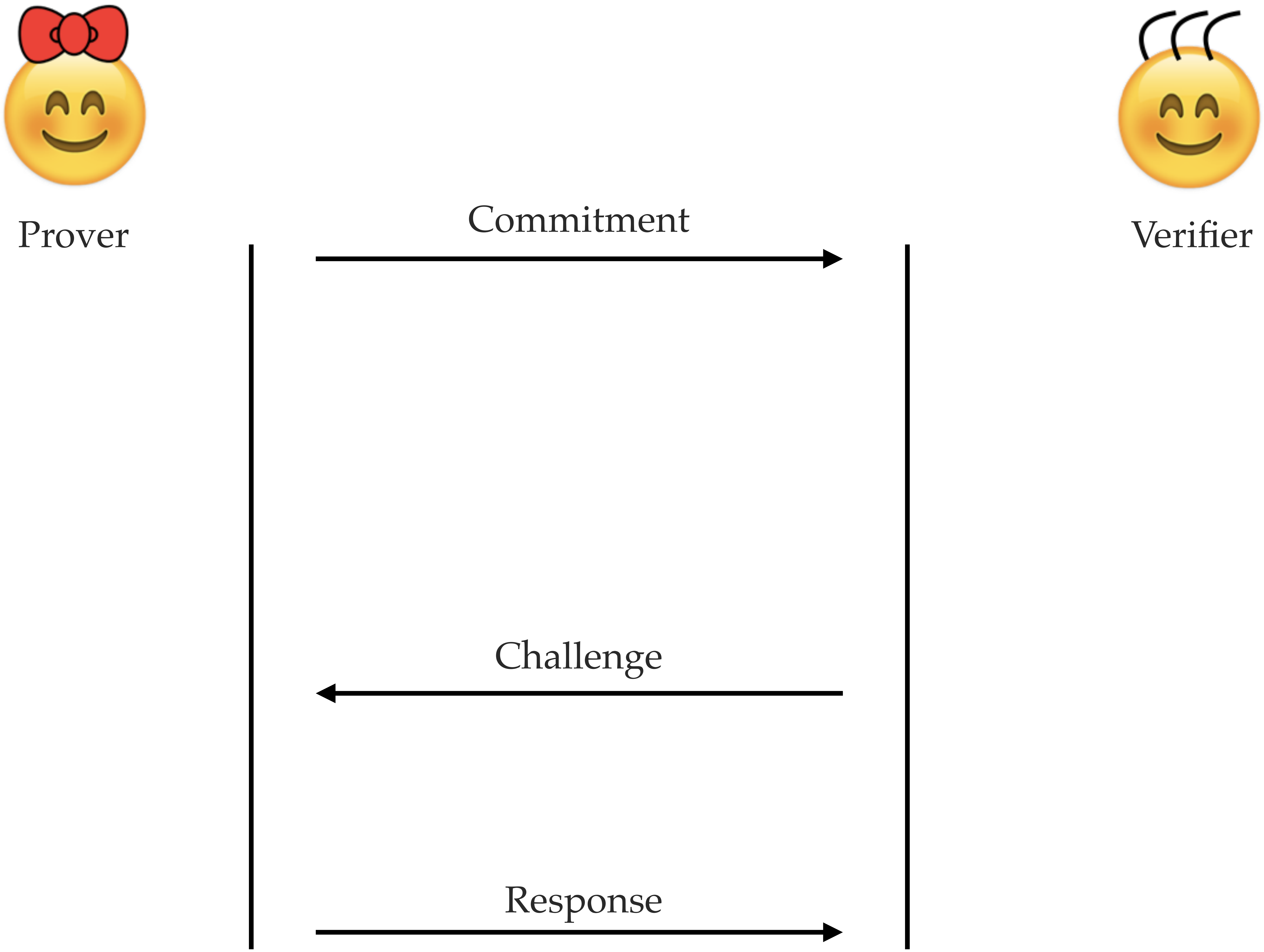


→ Properties that the underlying MPC model needs to have:

- $(N-1)$-private: the views of any $N-1$ parties do not reveal any information on x.
- Semi-honest: weak notion in MPC, but enough for the MPCitH application.

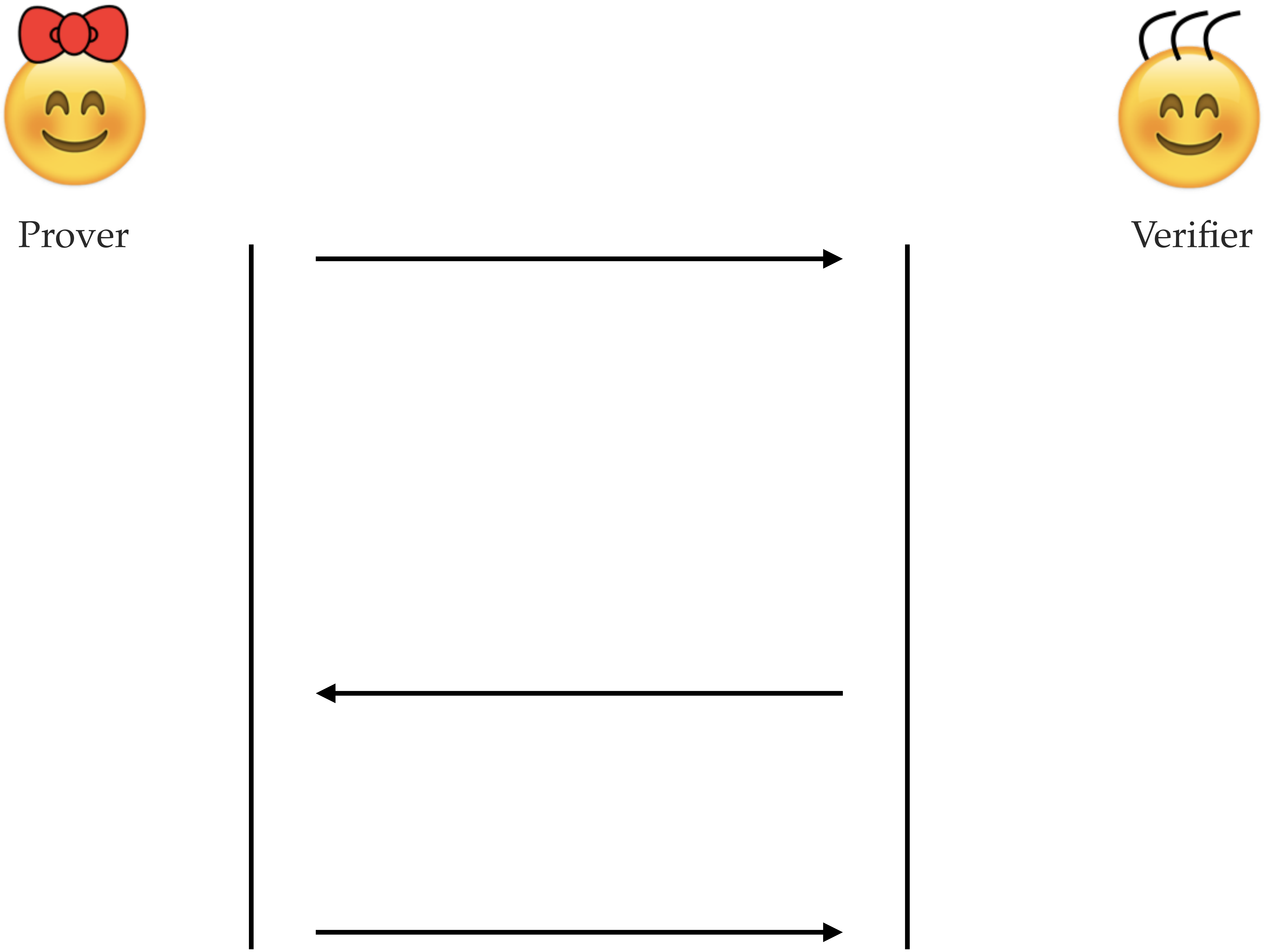
→ First instantiation: the PICNIC family (from symmetric primitives).

MPCitH identification scheme



MPCitH identification scheme

$\boxed{h = g^x}$



MPCitH identification scheme

$\boxed{h = g^x}$

Prover

Verifier

Computes $[[h]]_i = g^{[[x]]_i}$ ‘in his head’.

MPCitH identification scheme

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Prover

Verifier

$\mathrm{com}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)$

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Verifier

$\text{com}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)$ $\longrightarrow$

Computes $[[h]]_i = g^{[[x]]_i}$ 'in his head'.

Chooses $c \in [1; N]$.

$\xleftarrow{\;c\;}$

$\longrightarrow$

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$$\xrightarrow{\mathrm{com}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)}$$

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$$\xleftarrow{c}$$

$$\xrightarrow{[[x]]_i \text{ for all } i \neq c}$$

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Checks that $\prod_{i=1}^{N} [[h_i]] = h$.

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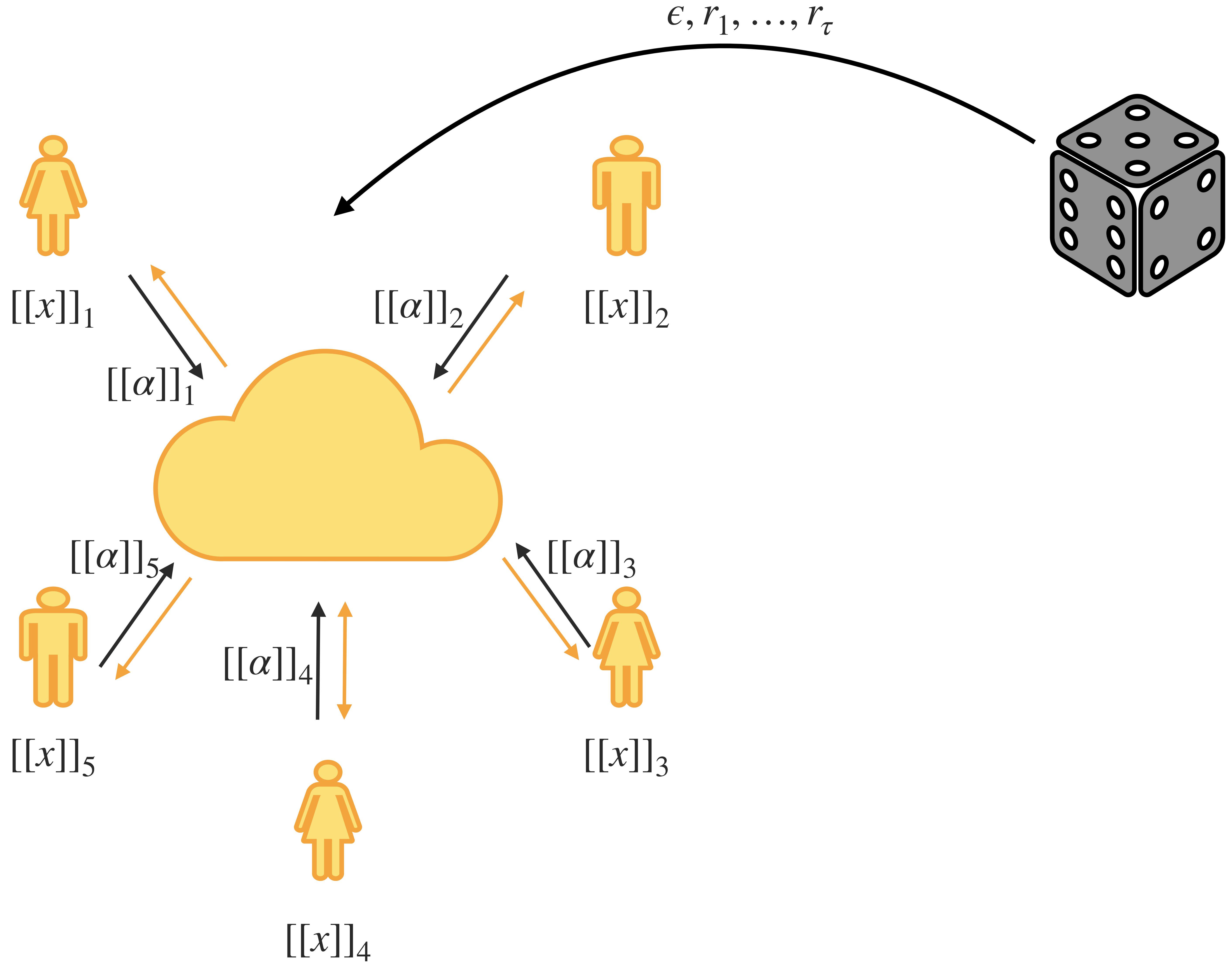
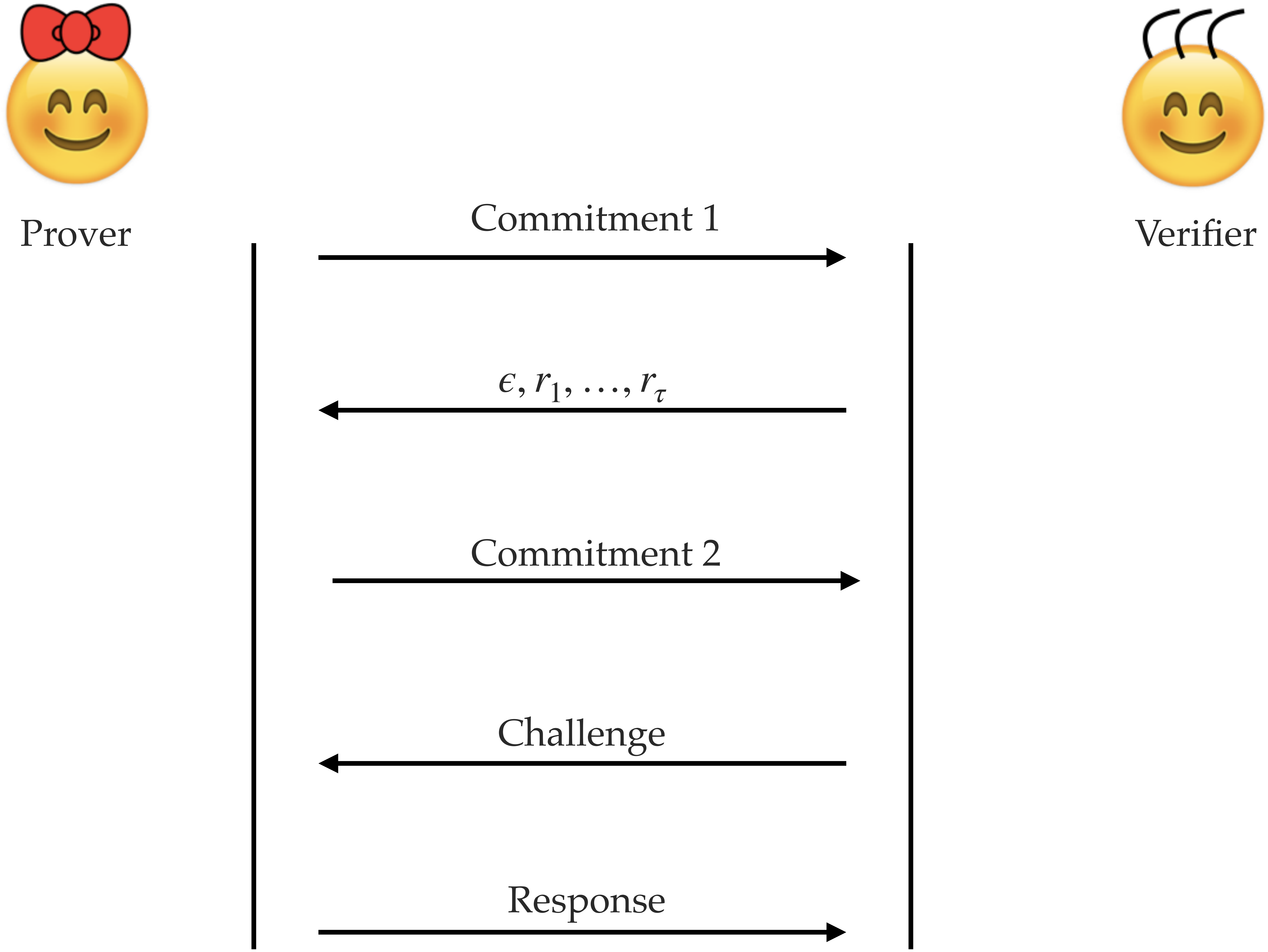
Checks that $[[h]]_i = g^{[[x]]_i}$ for all i.

Checks that $\prod_{i=1}^{N} [[h_i]] = h$. ✓

Broadcast model with oracle



A 5-pass protocol



Security properties

Completeness

If the statement is true, an honest prover is always able to convince an honest verifier.

Soundness

A dishonest prover cannot convince an honest verifier other than with a small probability.

Soundness

A dishonest prover cannot convince an honest verifier other than with a small probability.

2-Special soundness

Having obtained two valid transcripts with the same commitment and a different challenge, we can extract a solution for the underlying problem.

Zero-knowledge

Anyone observing the transcript (including the verifier) learns nothing other than the fact that the statement is true.

Security properties

→ Completeness

Security properties

→ Completeness ✓

Security properties

→ Completeness ✓

→ Soundness

Security properties

→ Completeness ✓

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Two ways of cheating:

→ Guessing an ϵ_i (input from oracle).

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Security properties

→ Completeness ✓

→ Soundness ✓

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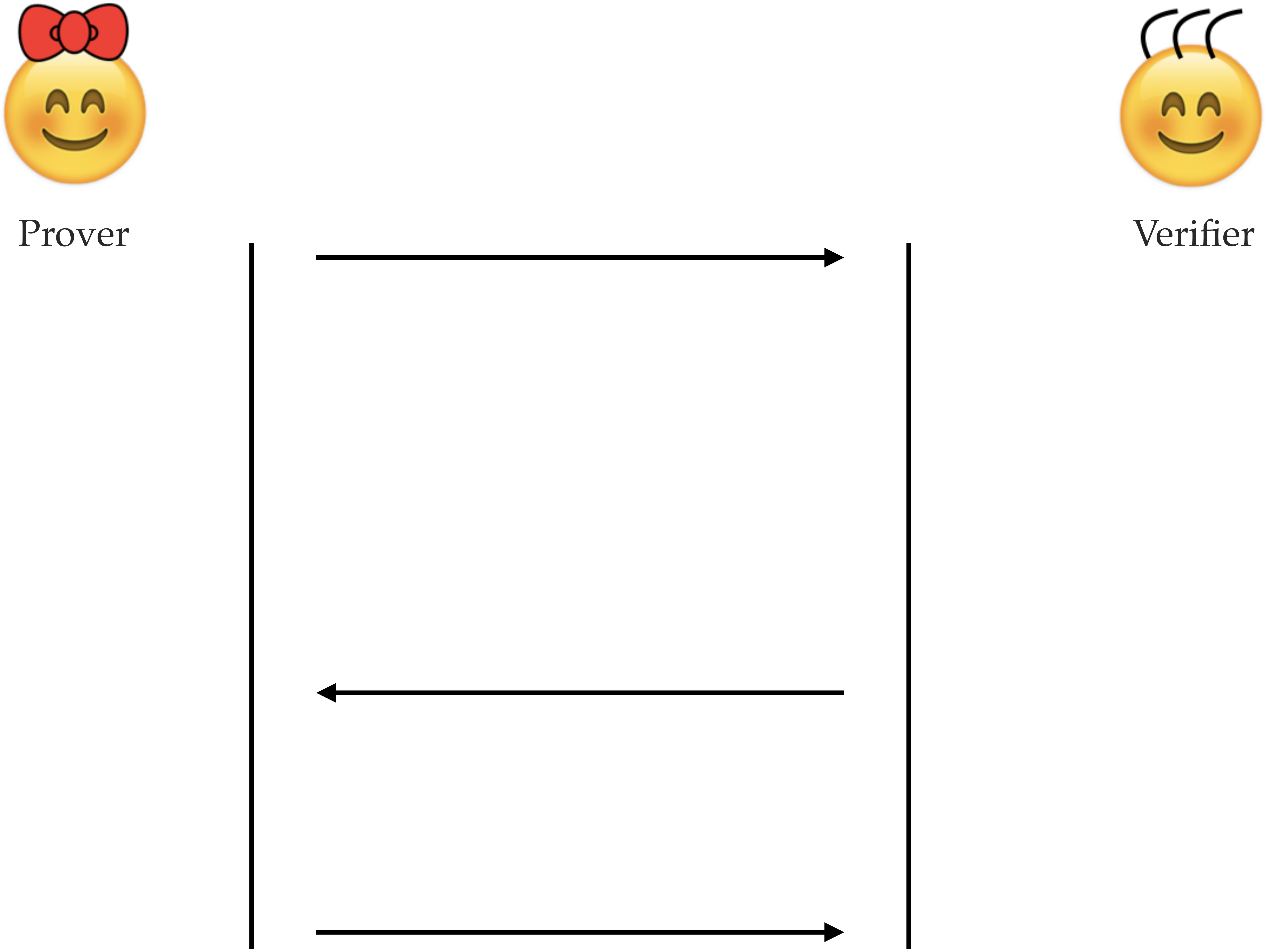
Malicious prover

$\boxed{h = g^x}$

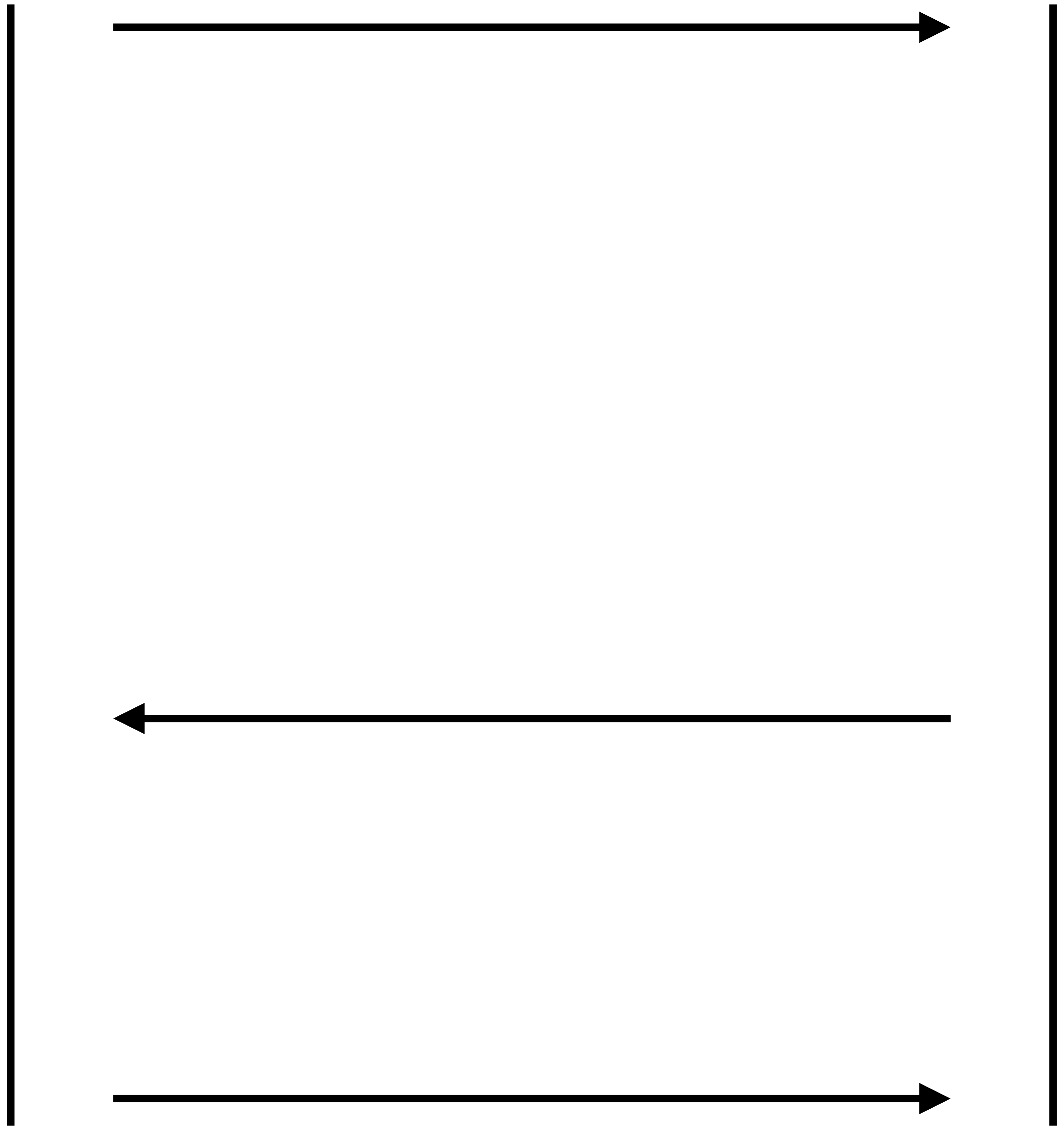


Malicious prover

$\boxed{h = g^x}$

Prover

Verifier



Chooses $[[x]]_i$ for $i \in [1; N-1]$.

Malicious prover

$\boxed{h = g^x}$

Prover

Verifier

Chooses $[[x]]_i$ for $i \in [1; N-1]$.

Computes $[[h]]_i = g^{[[x]]_i}$ for $i \in [1; N-1]$.

Computes $[[h]]_N = h / \prod_{i=1}^{N-1} [[h]]_i$.

Malicious prover

$\boxed{h = g^x}$

Prover

Verifier

Chooses $[[x]]_i$ for $i \in [1; N-1]$.

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Computes $[[h]]_N = h / \prod_{i=1}^{N-1} [[h]]_i$.

Cannot compute $[[x]]_N$ s.t. $[[h]]_N = g^{[[x]]_N}$ because the discrete log is hard.

! Not PQ (in this example)

Malicious prover

$\boxed{h = g^x}$

Prover

Verifier

$\xrightarrow{\mathrm{com}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)}$

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$[[x]]_i$ for all $i \neq c$ $\longrightarrow$

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← $\boxed{c = N}$

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Security properties

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Two ways of cheating:

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→ Zero-knowledge

Security properties

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As a result of the $(N-1)$-private property of the underlying MPC protocol.

Security properties

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MPC protocol

Multi-party computation

MPCitH identification scheme

MPC-in-the-Head

via the Fiat-Shamir transform

Digital signature

The Fiat-Shamir transform

The goal is to transform an interactive identification scheme into a digital signature scheme.

Instead of the prover choosing a challenge, the challenge is determined by the hash of the message and commitments.

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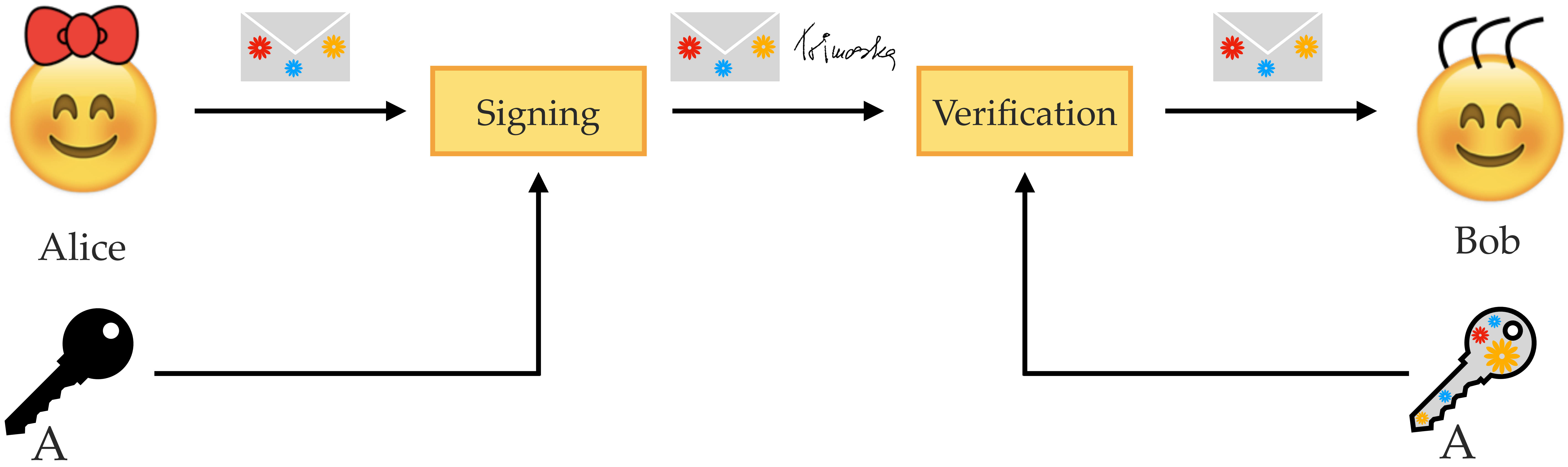

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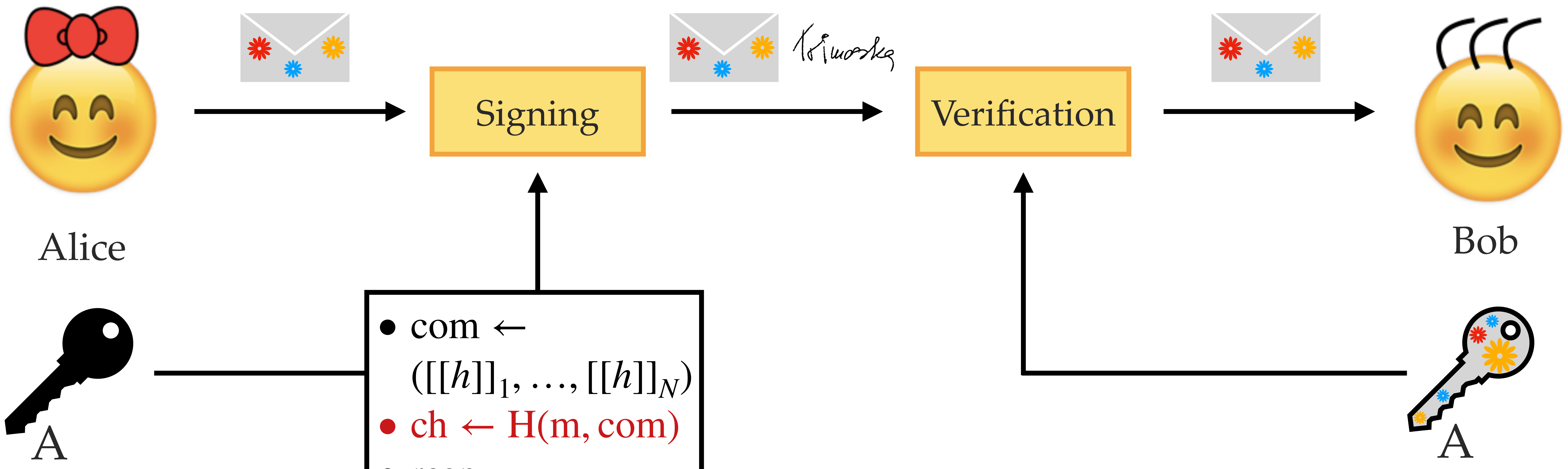

- com ← $([[h]]_1, \ldots, [[h]]_N)$
- ch ← H(m, com)
- resp ← $[[x]]_i$ for all $i \neq c$

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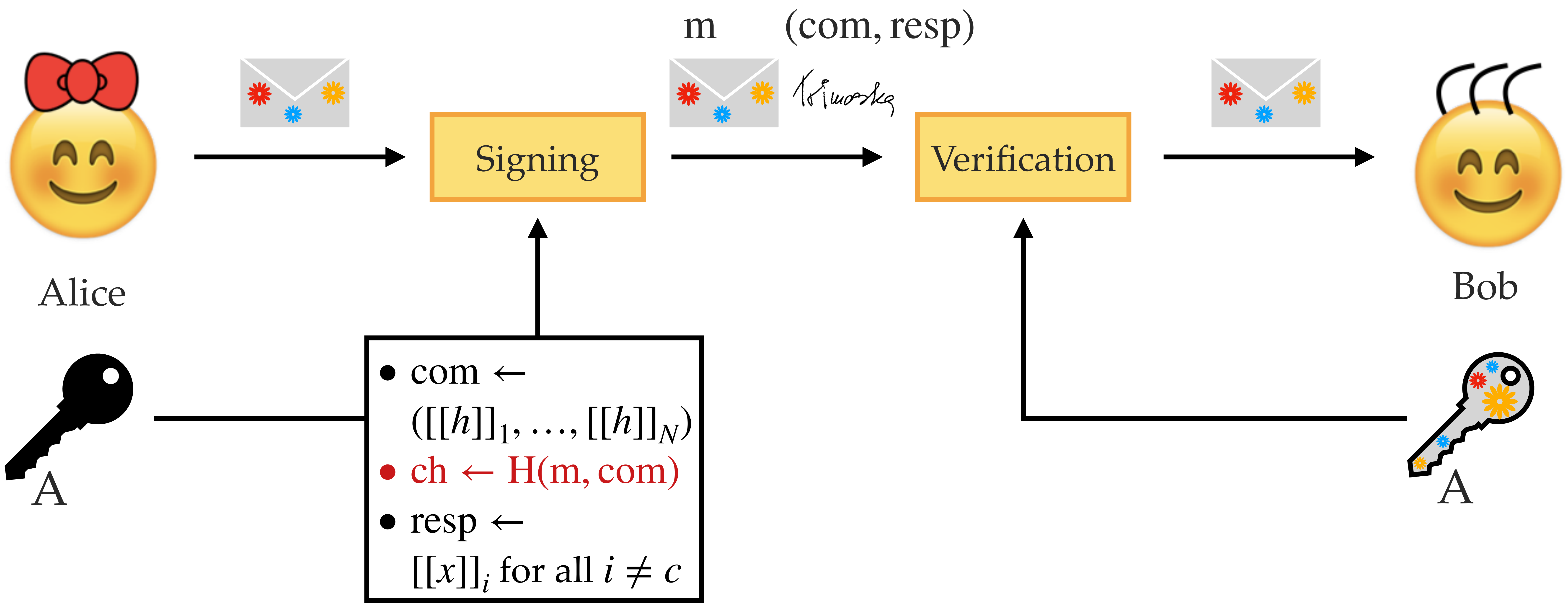

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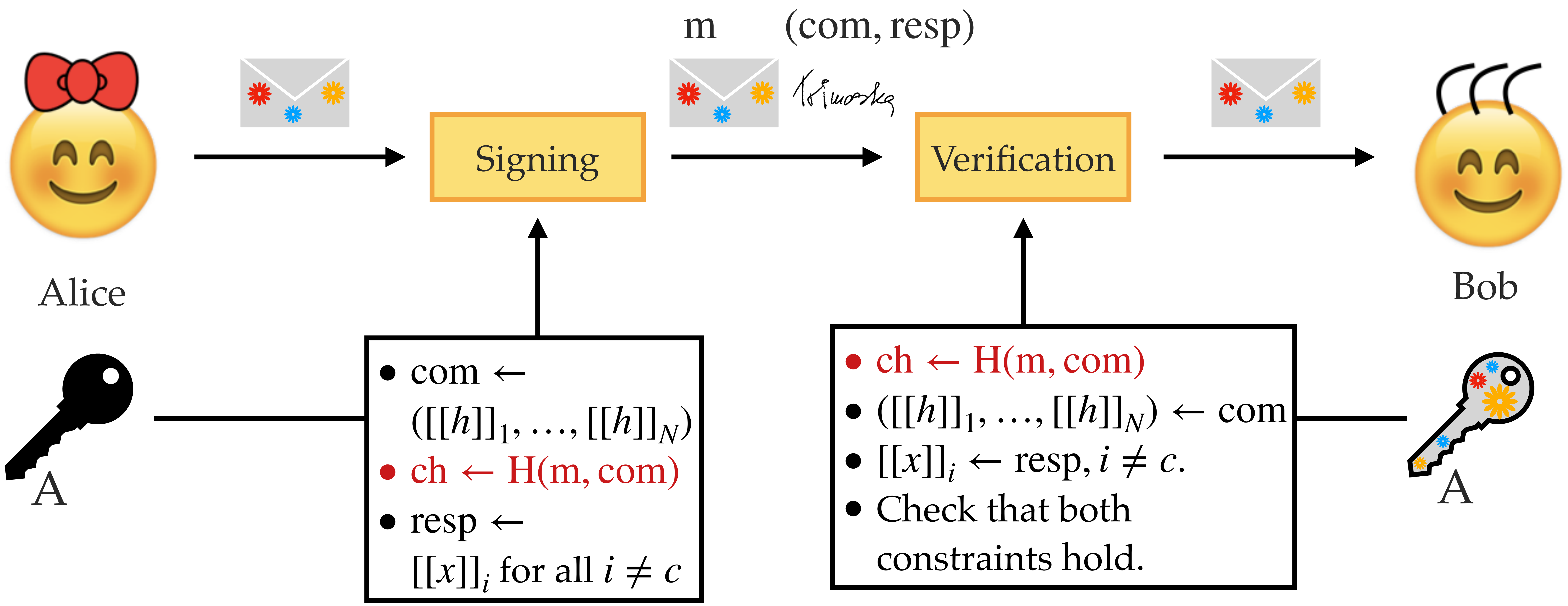

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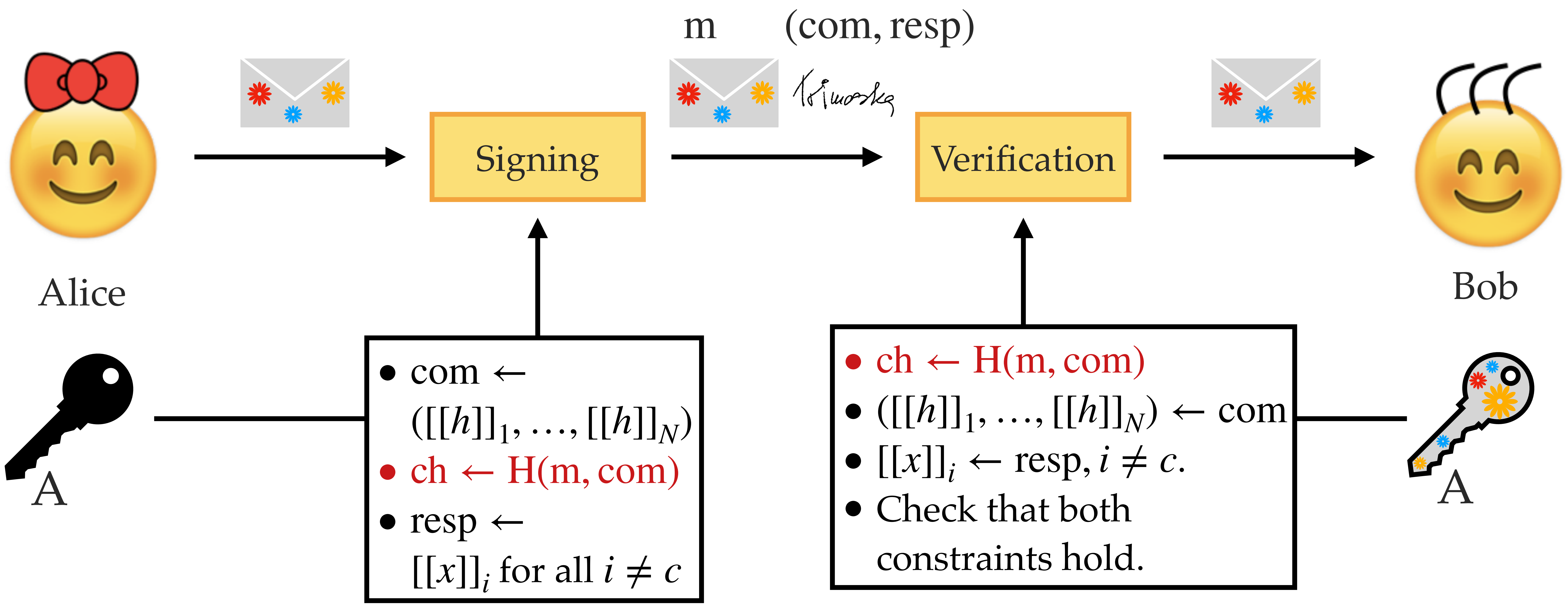


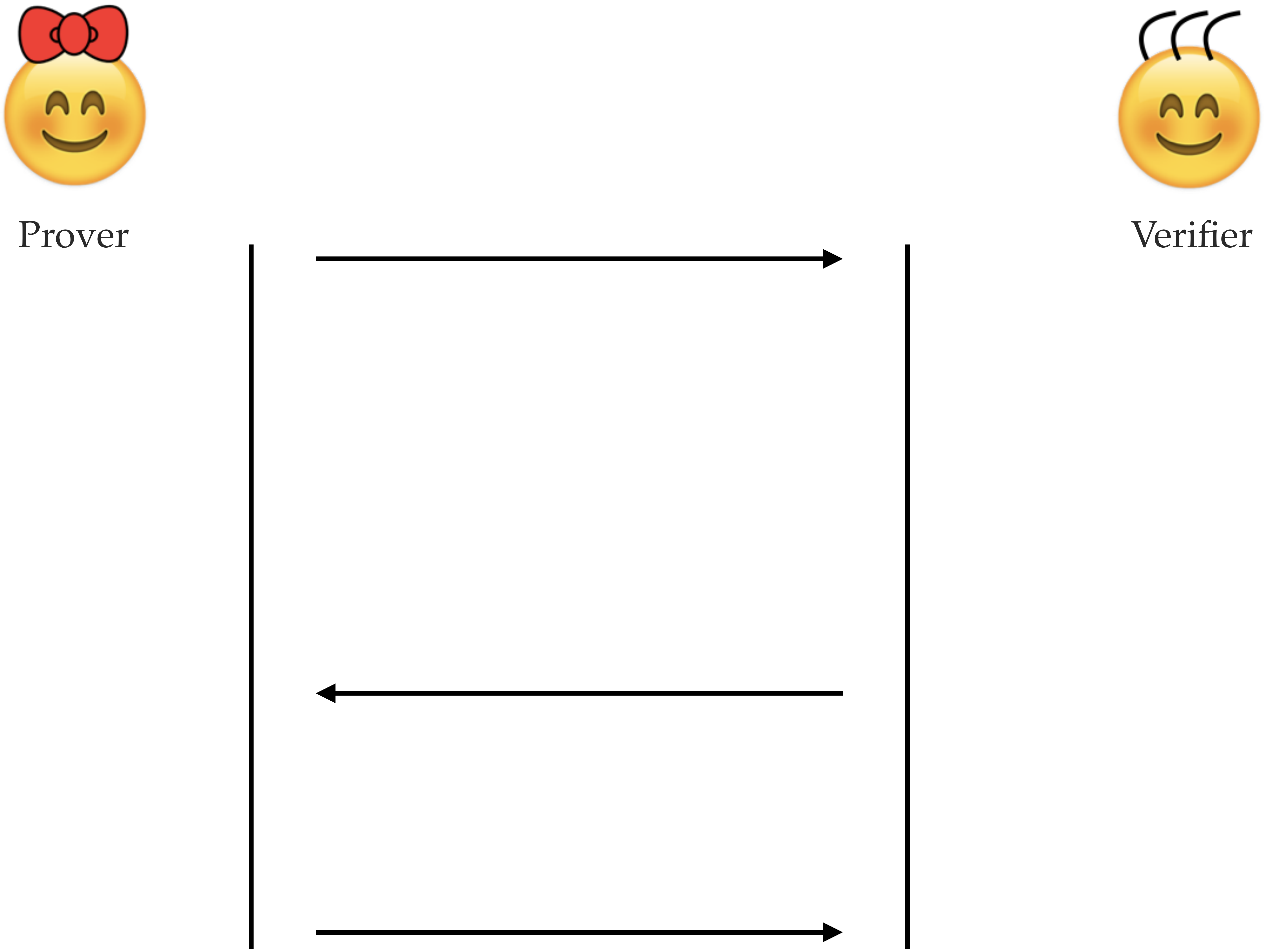
example: using the discrete log; showing a 3-pass (Sigma) protocol;

Optimizations

Optimizations

- Hashing the commitments
- Seed tree
- Hypercube
- Threshold

Hashing the commitments



Hashing the commitments

Prover

Verifier

Computes $[[h]]_i = g^{[[x]]_i}$ 'in his head'.

Hashing the commitments

Prover

Verifier

$H = \mathrm{Hash}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)$

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Hashing the commitments

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Chooses $c \in [1; N]$.

$\xleftarrow{\;c\;}$

$\longrightarrow$

Hashing the commitments

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Chooses $c \in [1; N]$.

← c

$[[x]]_i$ for all $i \neq c$ →

Hashing the commitments

Prover

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$H = \mathrm{Hash}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)$ $\longrightarrow$

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Chooses $c \in [1; N]$.

$\longleftarrow$ c

Computes $H' = \mathrm{Hash}([[h]]_1, \ldots, [[h]]_N)$.

$[[x]]_i$ for all $i \neq c$ $\longrightarrow$

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← c

$[[x]]_i$ for all $i \neq c$ →

Computes $H' = \text{Hash}([[h]]_1, \ldots, [[h]]_N)$.

Checks that $H = H'$.

Hashing the commitments

Prover

Verifier

$H = \text{Hash}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)$ →

Computes $[[h]]_i = g^{[[x]]_i}$ 'in his head'.

Chooses $c \in [1; N]$.

Computes $[[h]]_i = g^{[[x]]_i}$ for $i \neq c$.

← c

Computes $H' = \text{Hash}([[h]]_1, \ldots, [[h]]_N)$.

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$[[x]]_i$ for all $i \neq c$ →

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$H = \mathrm{Hash}([[h]]_1, [[h]]_2, \ldots, [[h]]_N)$ →

Chooses $c \in [1; N]$.

← c

$[[x]]_i$ for all $i \neq c$ →

Computes $[[h]]_i = g^{[[x]]_i}$ for $i \neq c$.

Computes $[[h]]_c = h / \prod_{i=1}^{N} [[h_i]]$, for $i \neq c$.

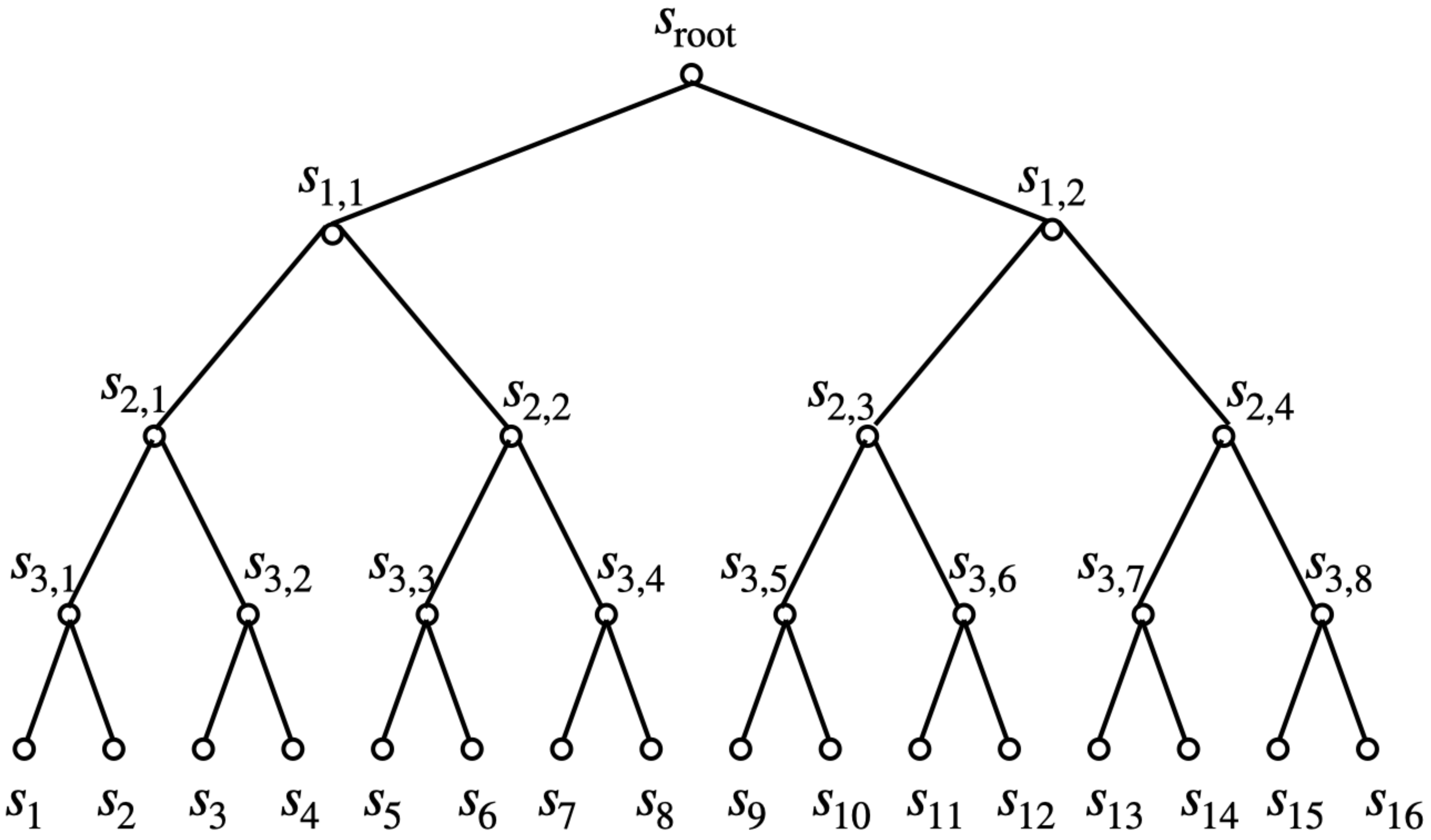
Computes $H' = \mathrm{Hash}([[h]]_1, \ldots, [[h]]_N)$.

Checks that $H = H'$.

Seed tree

Let $N = 2^t$.

We expand $[[x]]_1, \ldots, [[x]]_N$ from a root seed (adjust $[[x]]_N$ so that we get a sharing of x).

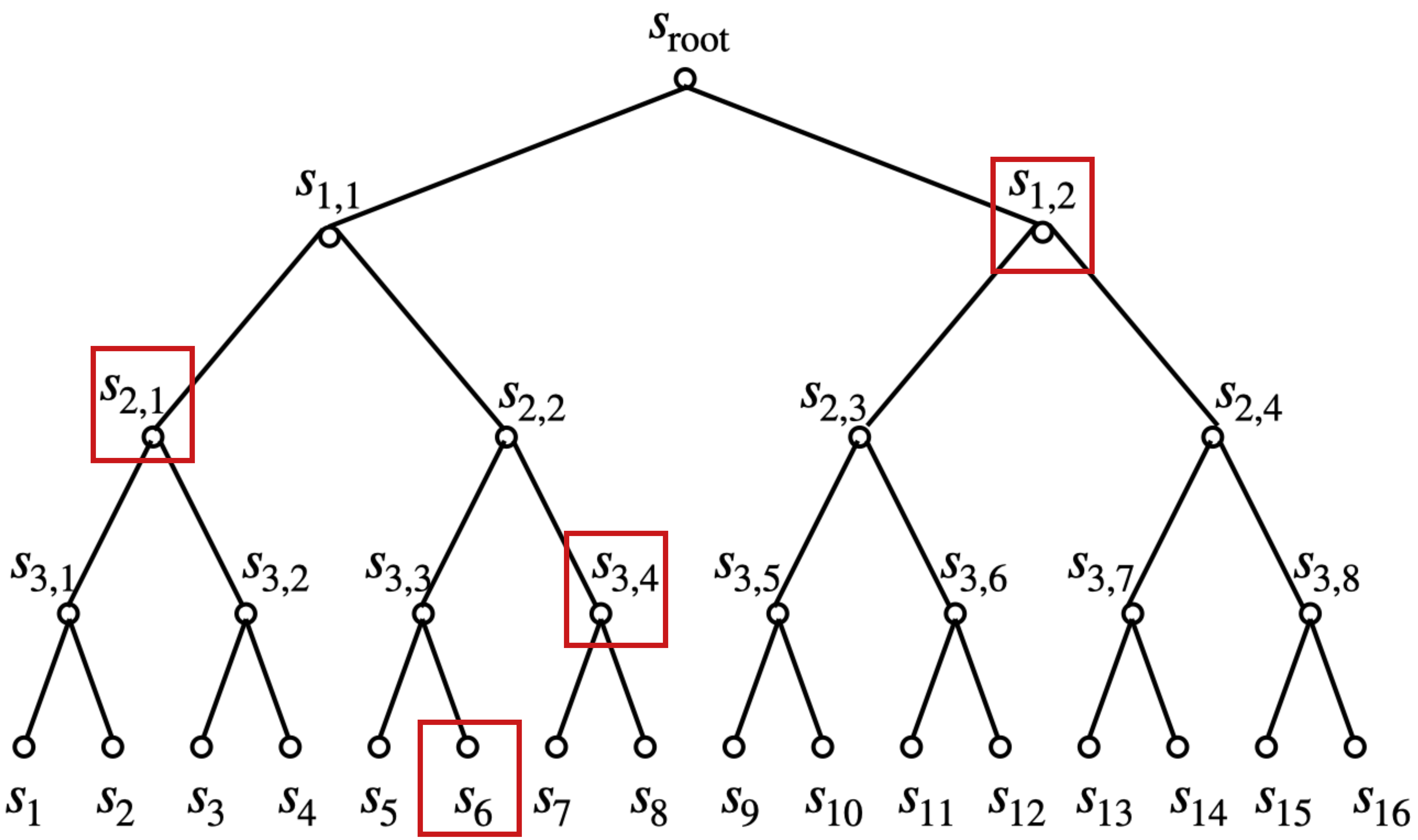


Seed tree

Let $N = 2^t$.

We expand $[[x]]_1, \ldots, [[x]]_N$ from a root seed (adjust $[[x]]_N$ so that we get a sharing of x).

$\longrightarrow c = 5$



Hypercube

A technique to turn one MPC instance (simulation) of N^D into D instances of N parties.

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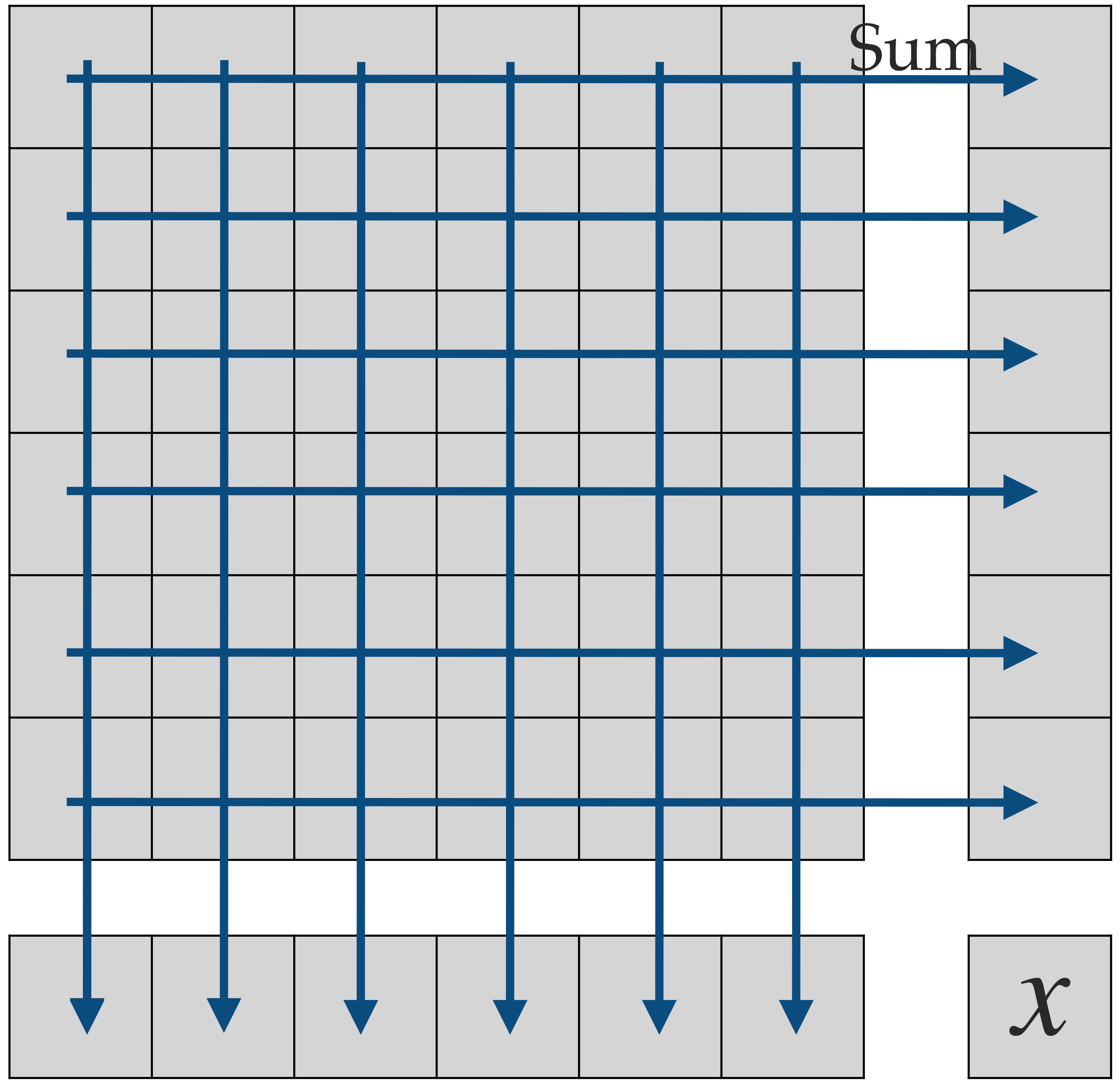
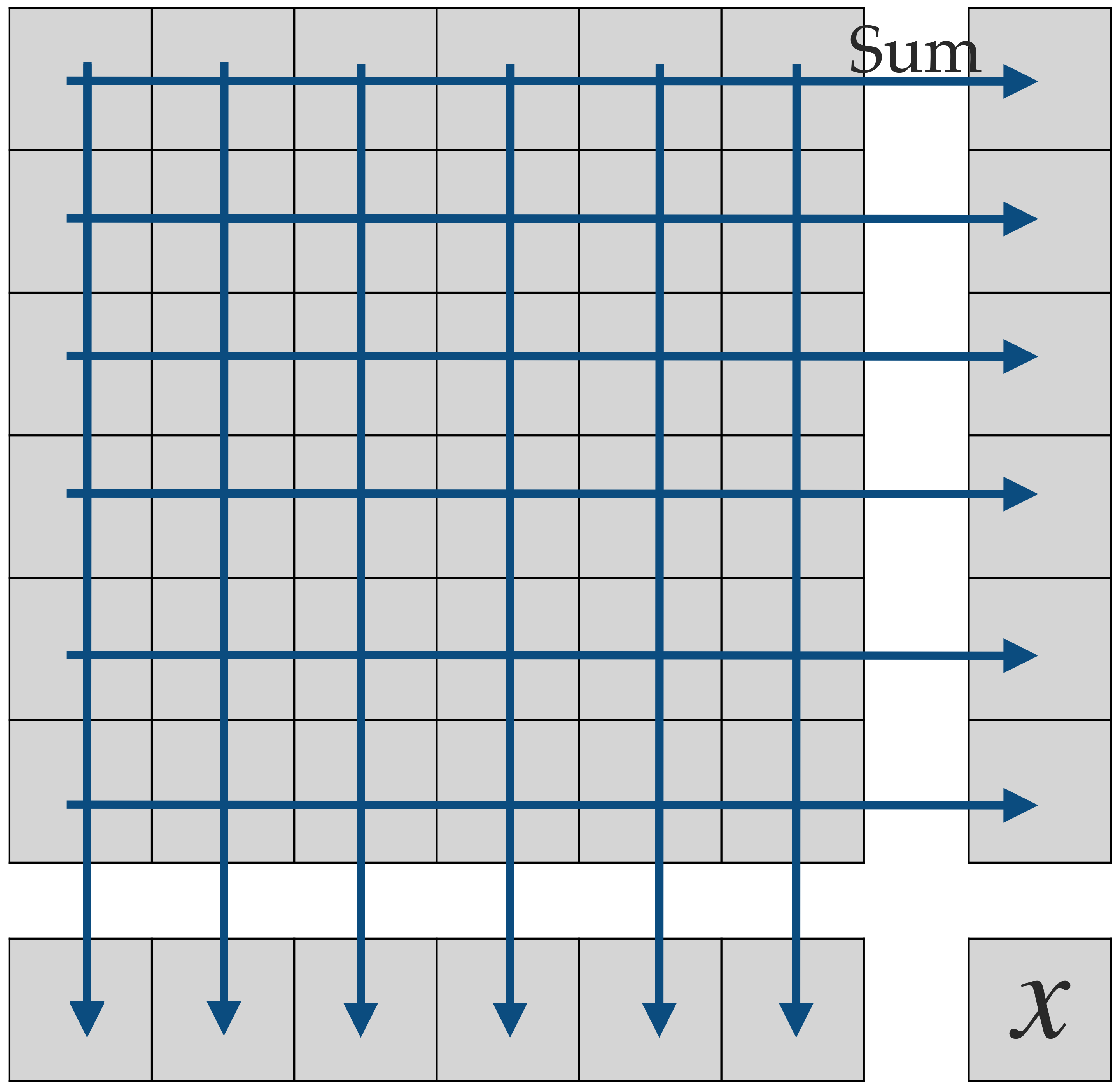


Hypercube

A technique to turn one MPC instance (simulation) of N^D into D instances of N parties.

<u>MPC 1:</u>
A $\sqrt{N}$-party protocol with shares corresponding the sums of column entries.

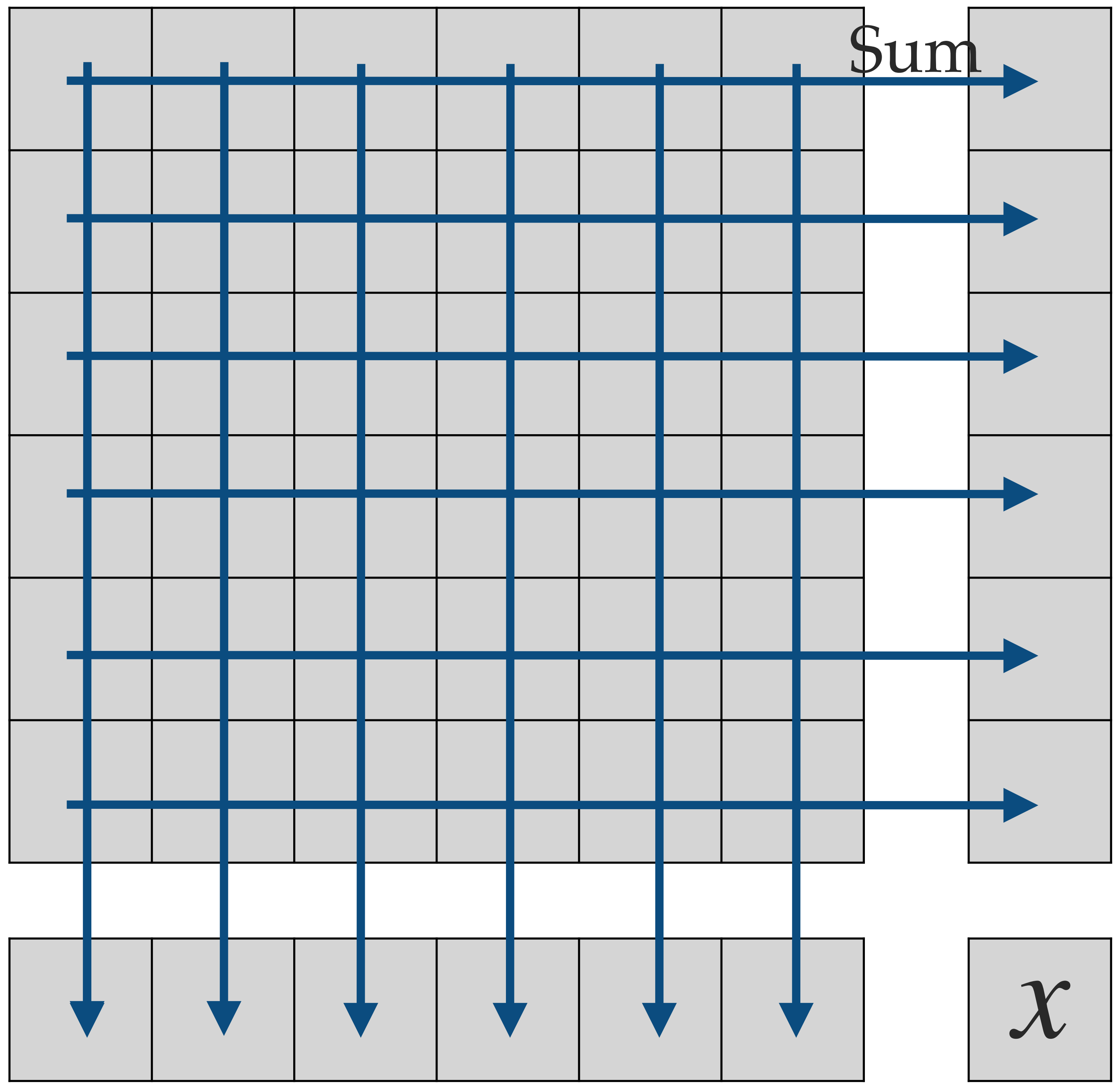


Hypercube

A technique to turn one MPC instance (simulation) of N^D into D instances of N parties.

<u>MPC 1:</u>
A $\sqrt{N}$-party protocol with shares corresponding the sums of column entries.

<u>MPC 2:</u>
A $\sqrt{N}$-party protocol with shares corresponding the sums of row entries.



Hypercube

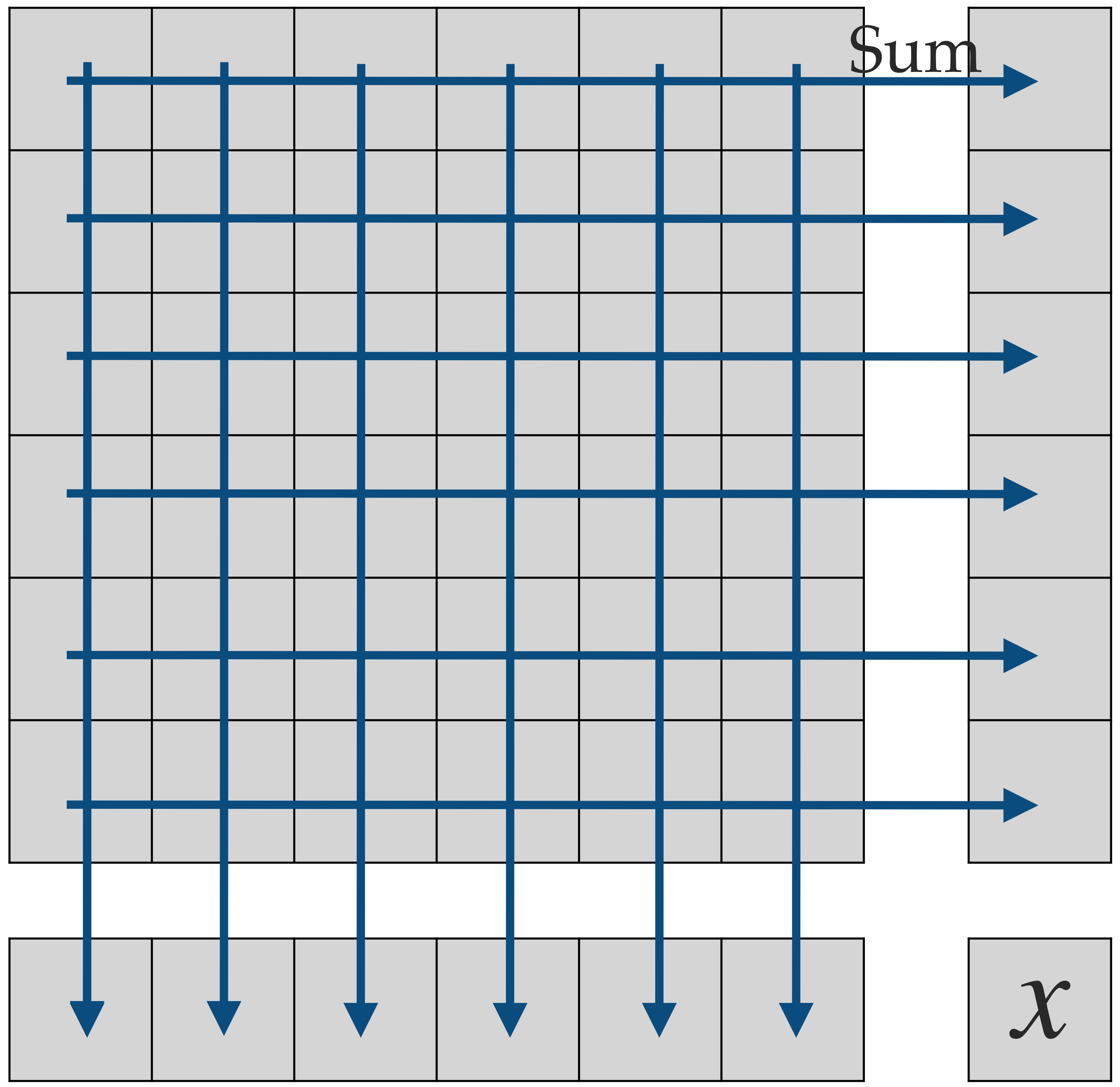
A technique to turn one MPC instance (simulation) of N^D into D instances of N parties.

MPC 1:
A $\sqrt{N}$-party protocol with shares corresponding the sums of column entries.

MPC 2:
A $\sqrt{N}$-party protocol with shares corresponding the sums of row entries.

Probability of cheating in both protocols:

$$\left(\frac{1}{\sqrt{N}}\right)^2$$



Hypercube

A technique to turn one MPC instance (simulation) of N^D into D instances of N parties.

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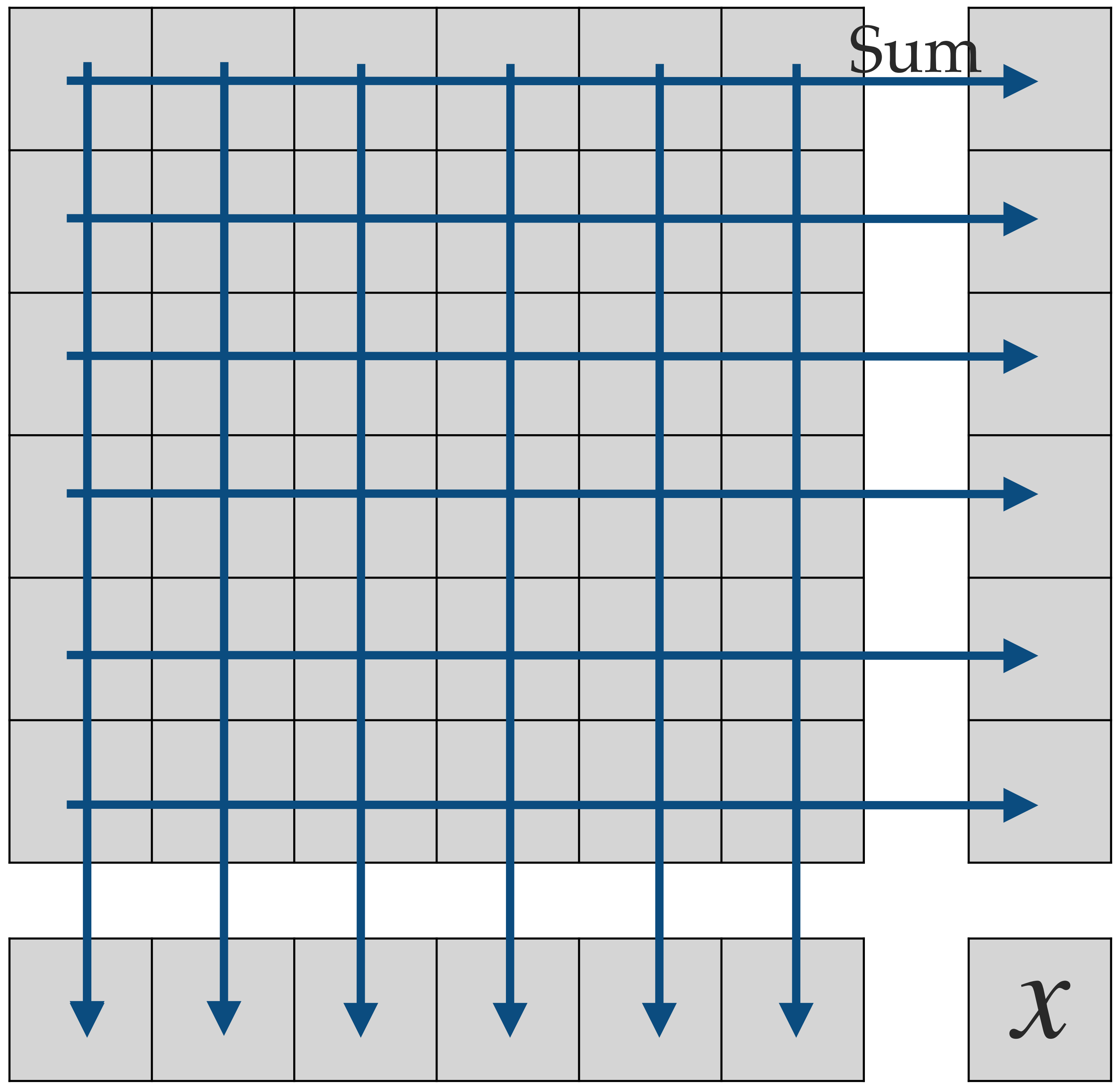


→ Probability of cheating in both protocols:

$$\left(\frac{1}{\sqrt{N}}\right)^2$$

→ $\frac{1}{N}$ (same as before!)

Hypercube

Going to higher dimensions

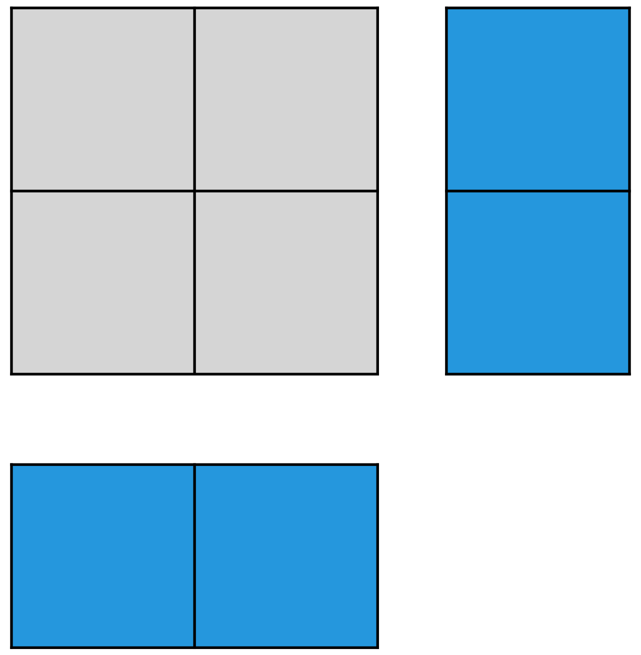
→ Best trade-off: take $N = 2$.

Hypercube

Going to higher dimensions

→ Best trade-off: take $N = 2$.

Example. $D = 2$

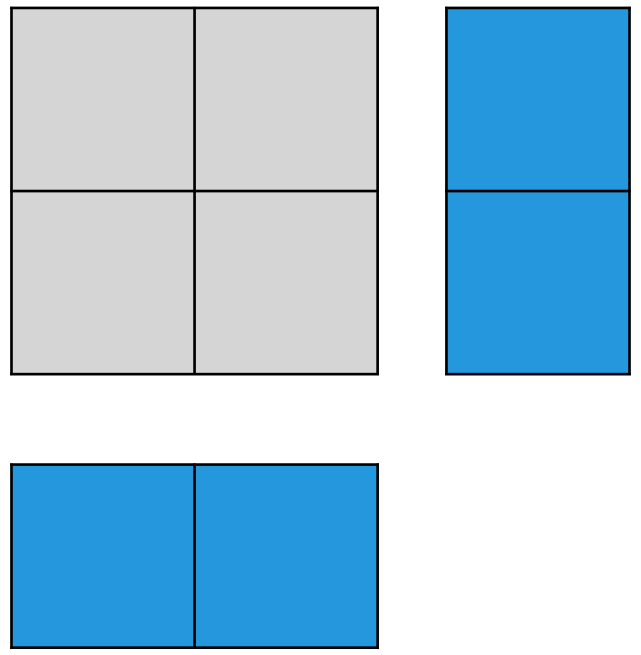


Hypercube

Going to higher dimensions

⟶ Best trade-off: take $N = 2$.

Example. $D = 2$



No difference between the two approaches.

Hypercube

Going to higher dimensions

Which shares participate in which MPC instance??

For k in $[1 \dots D]$
  MPC k:
  For j in [1,2]
    Take shares whose k-th coordinate is j.

Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 3$

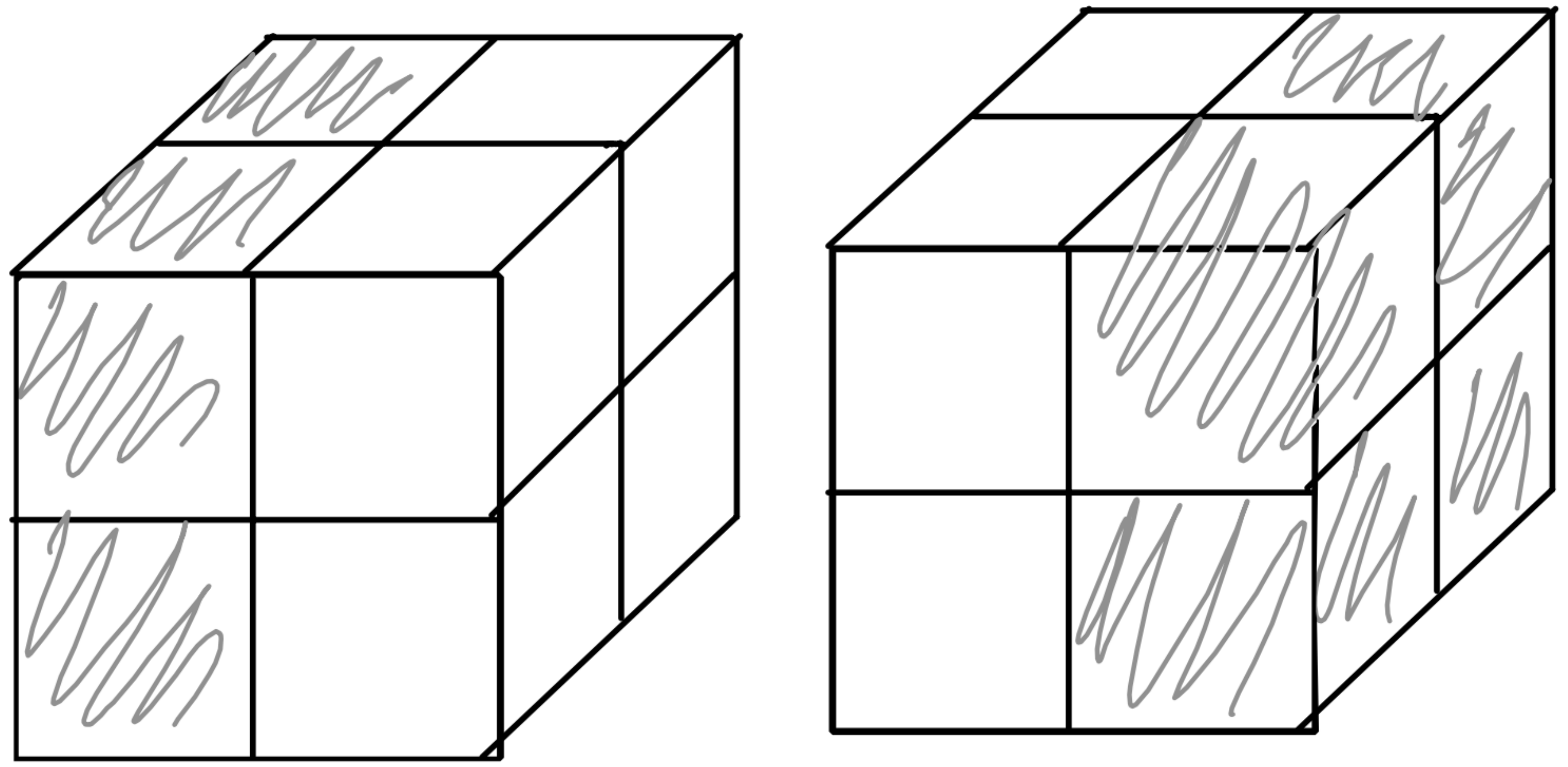
Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 3$

<u>MPC 1</u>



(1,1) (1,2)

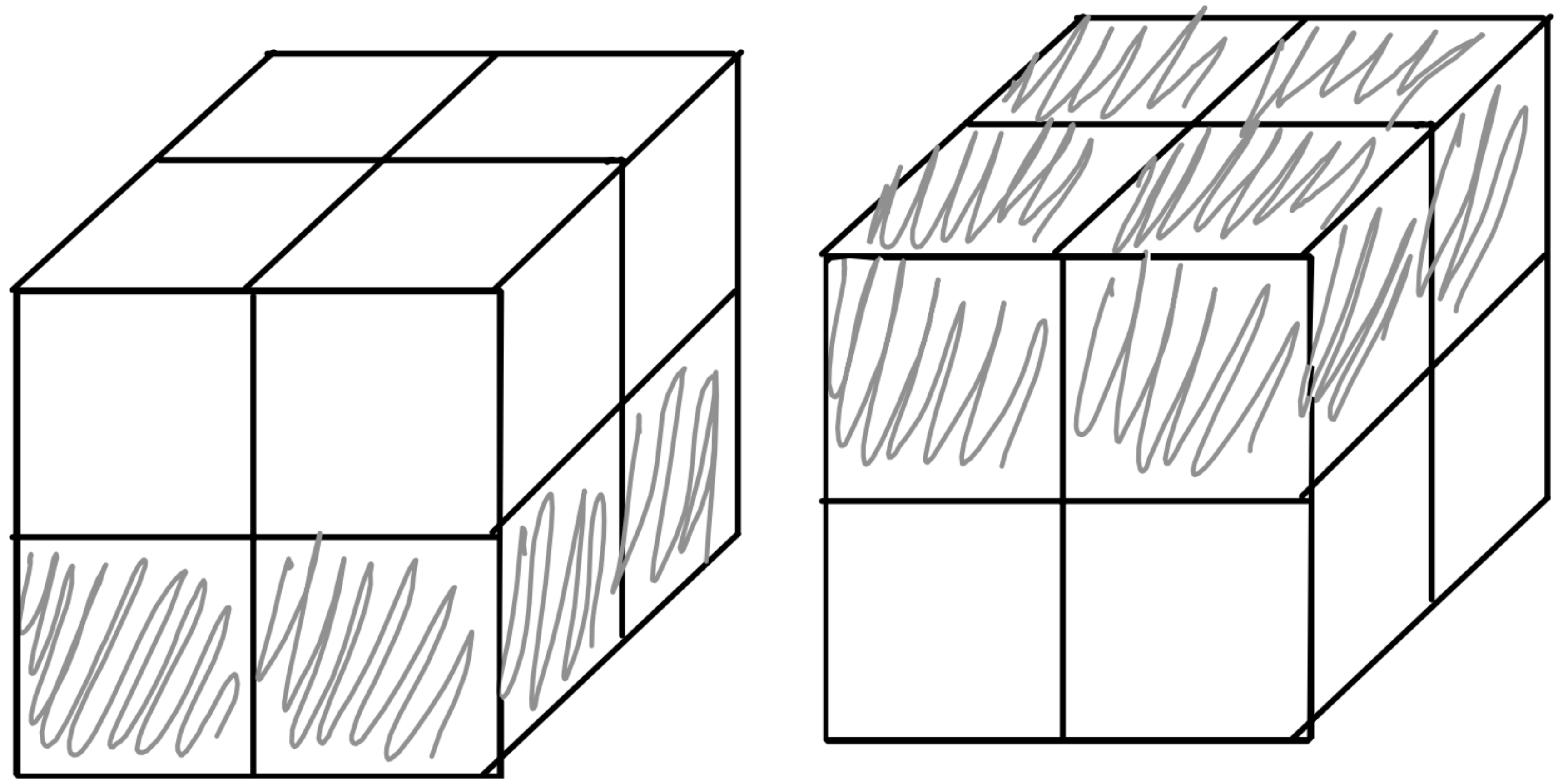
Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

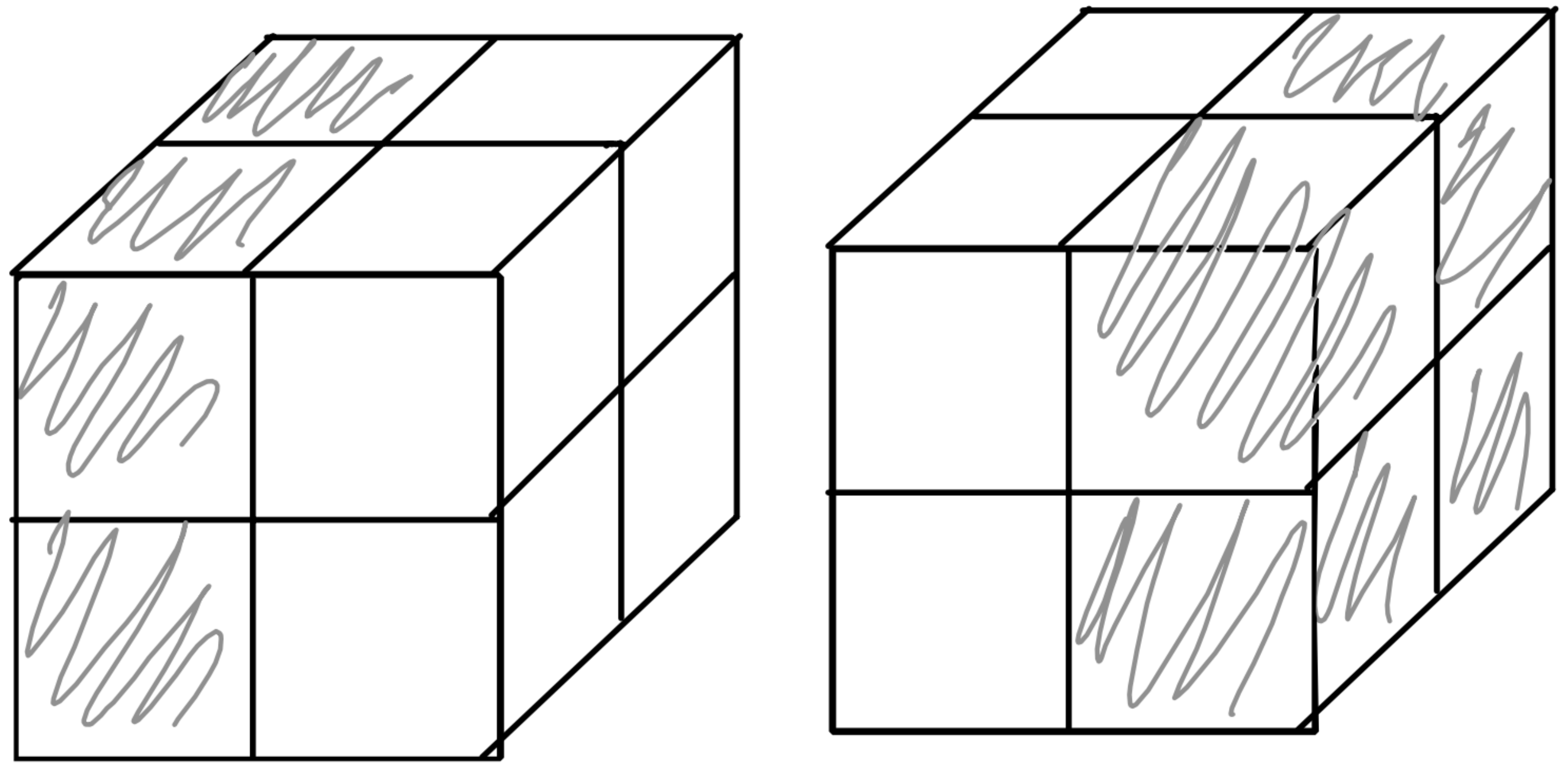
Example. $D = 3$

MPC 2



(2,1) (2,2)

MPC 1



(1,1) (1,2)

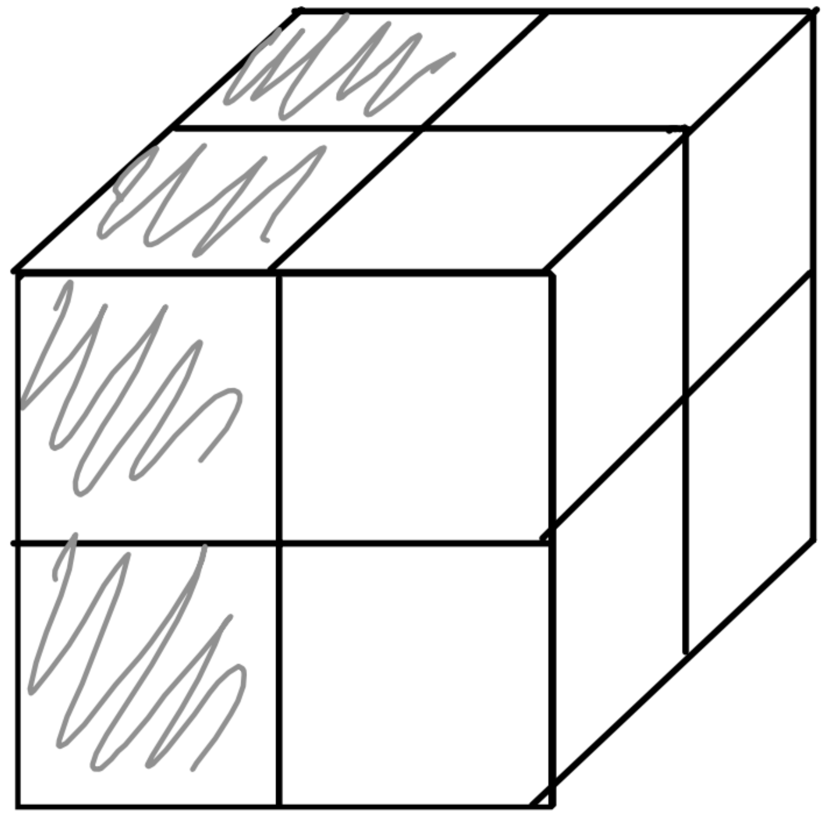
Hypercube

Take shares whose k-th coordinate is j.

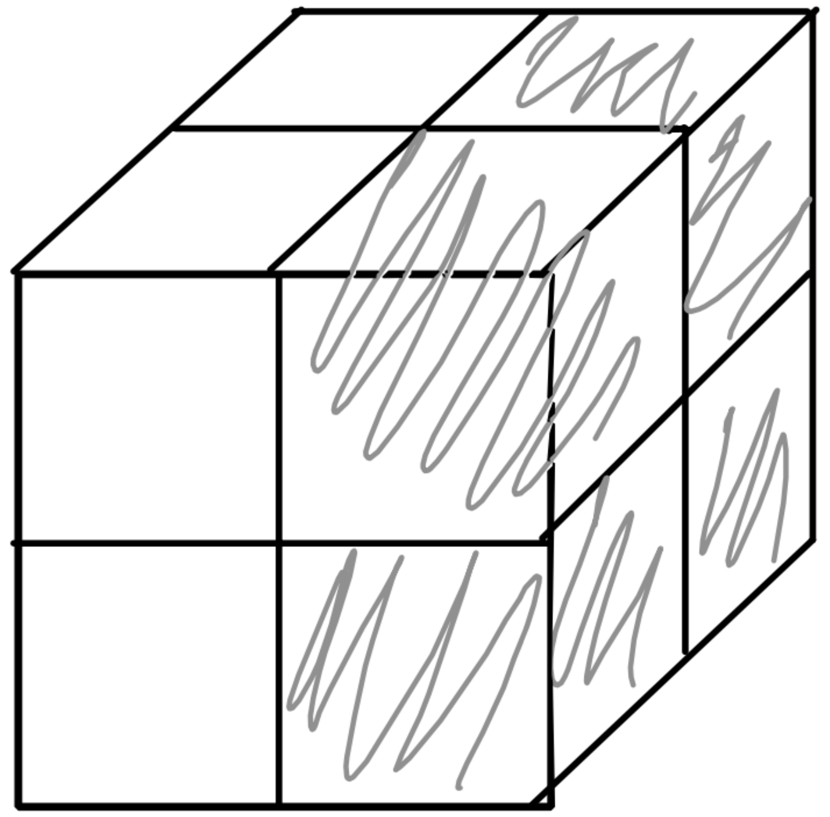
Going to higher dimensions

Example. $D = 3$

MPC 1

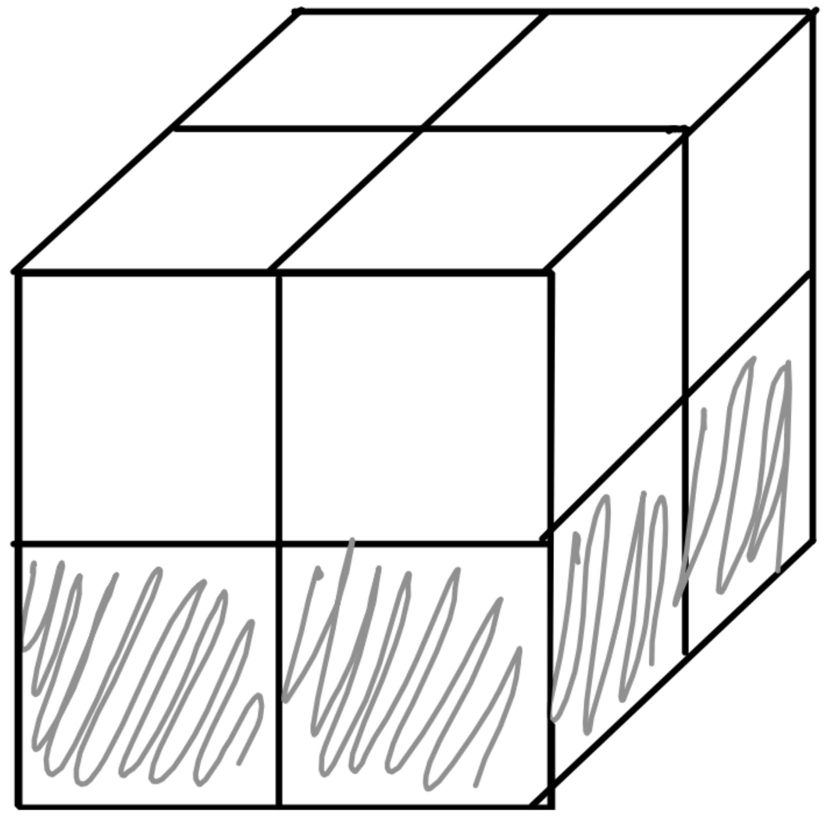


(1,1)

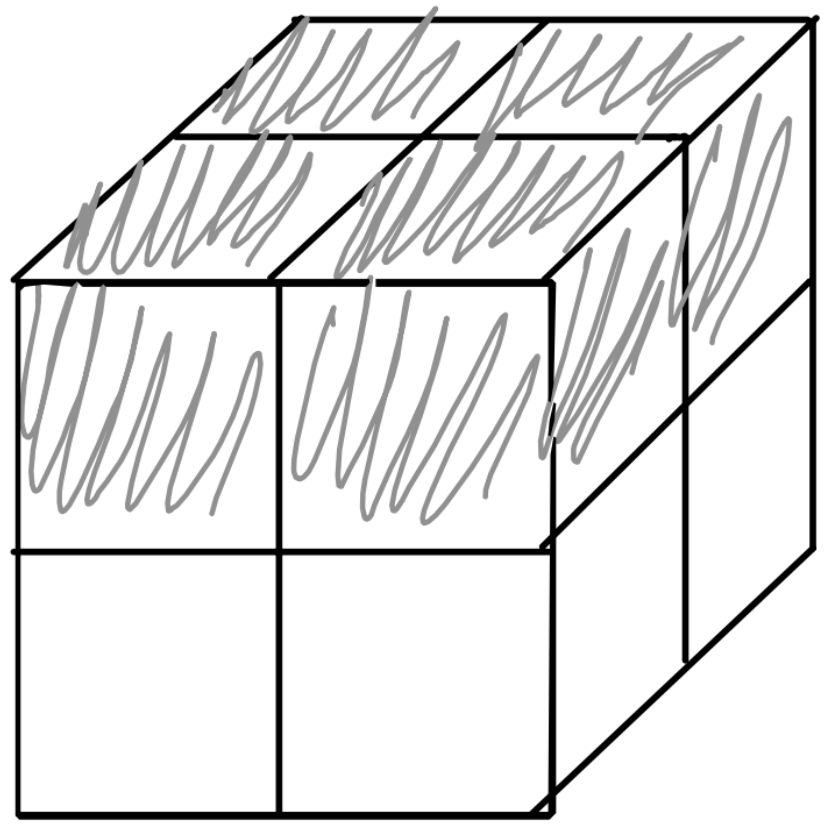


(1,2)

MPC 2

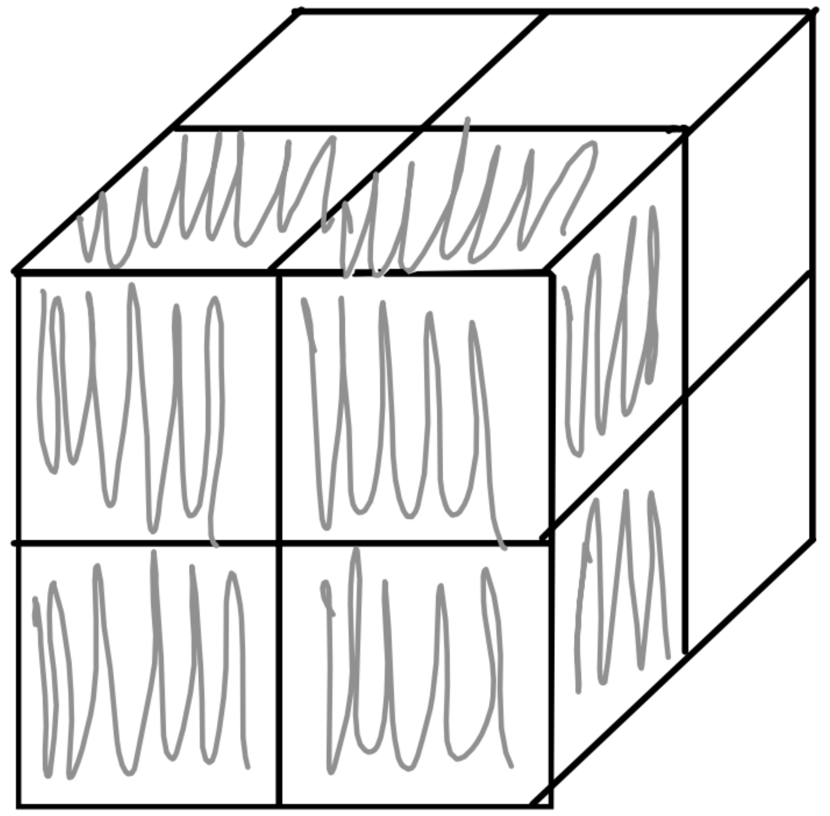


(2,1)

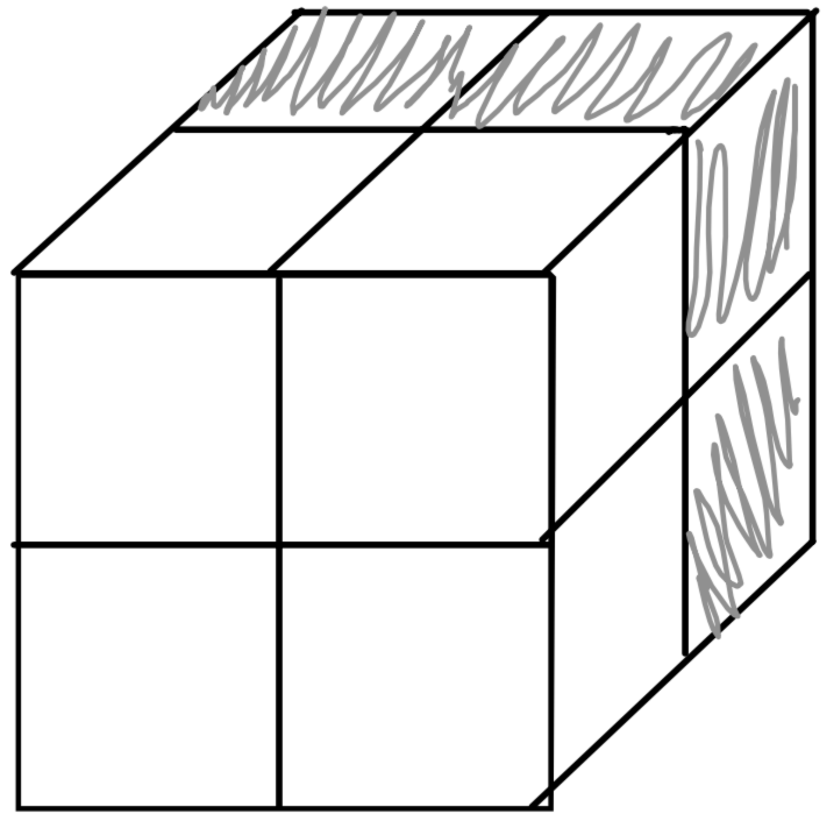


(2,2)

MPC 3



(3,1)



(3,2)

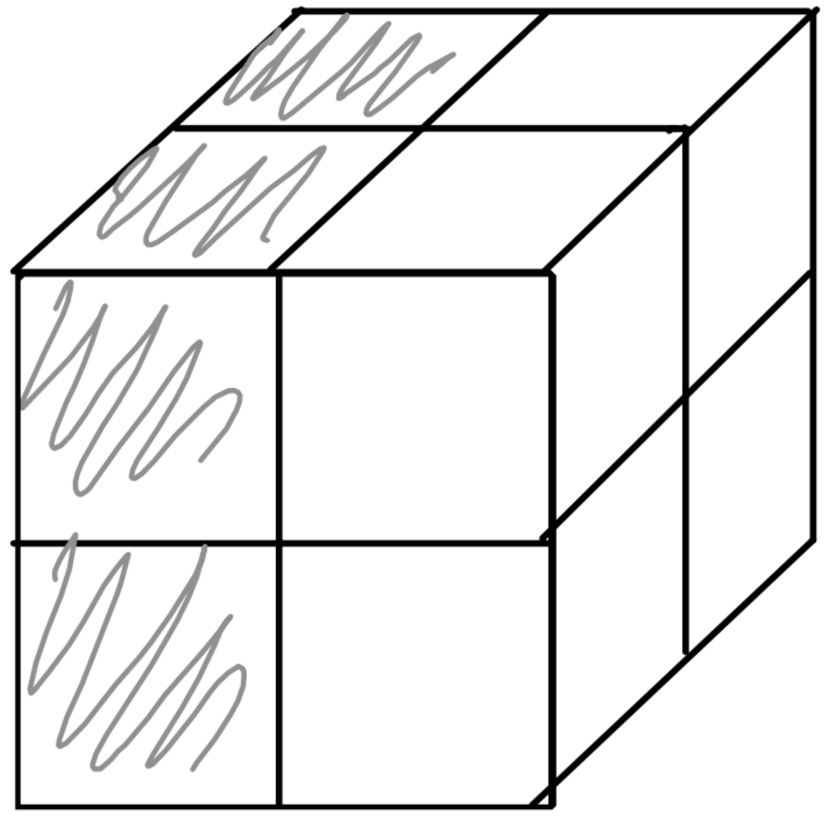
Hypercube

Take shares whose k-th coordinate is j.

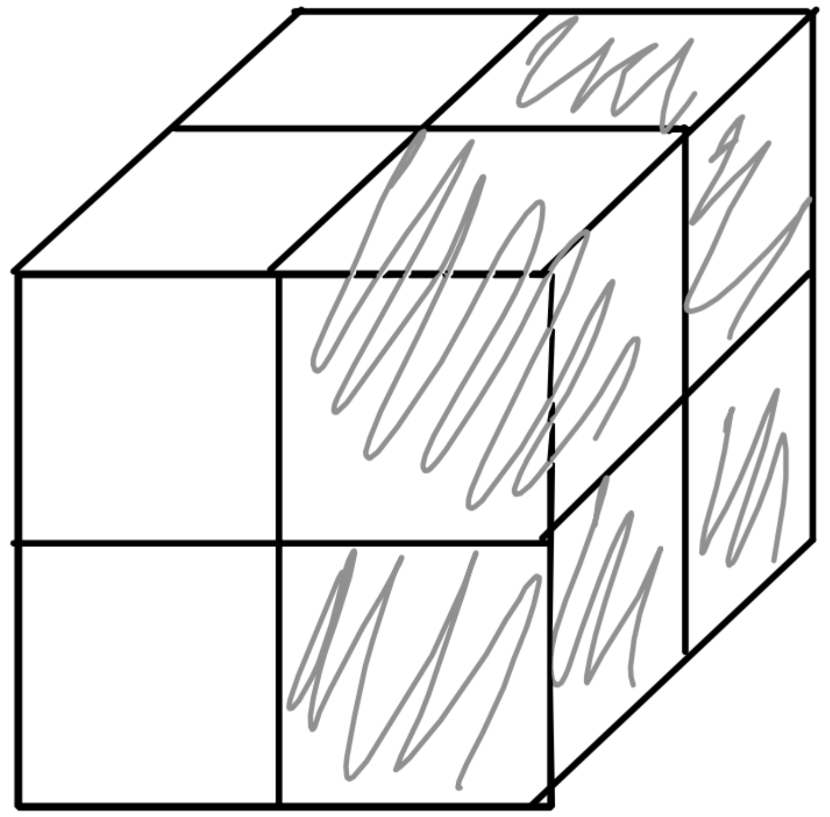
Going to higher dimensions

Example. $D = 3$

MPC 1



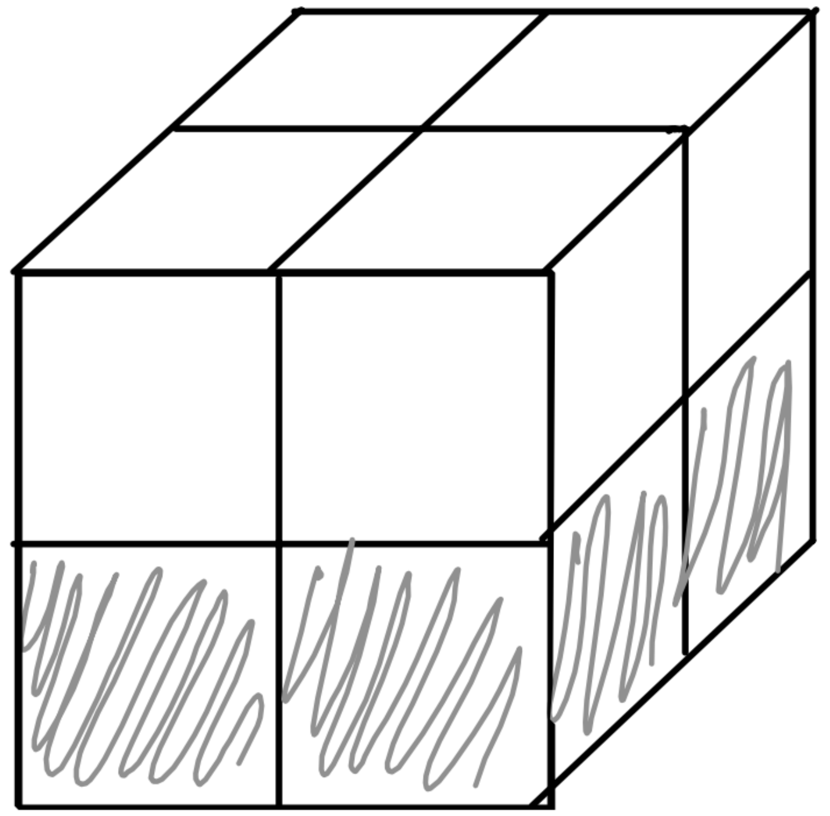
(1,1)



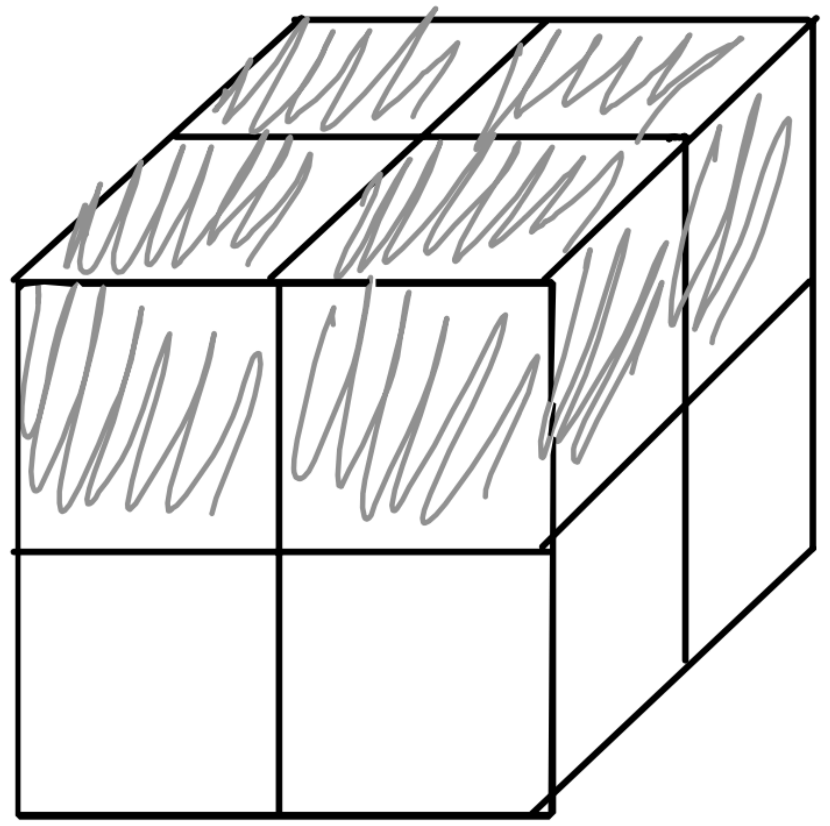
(1,2)

3 MPC instance of 2 main parties.

MPC 2

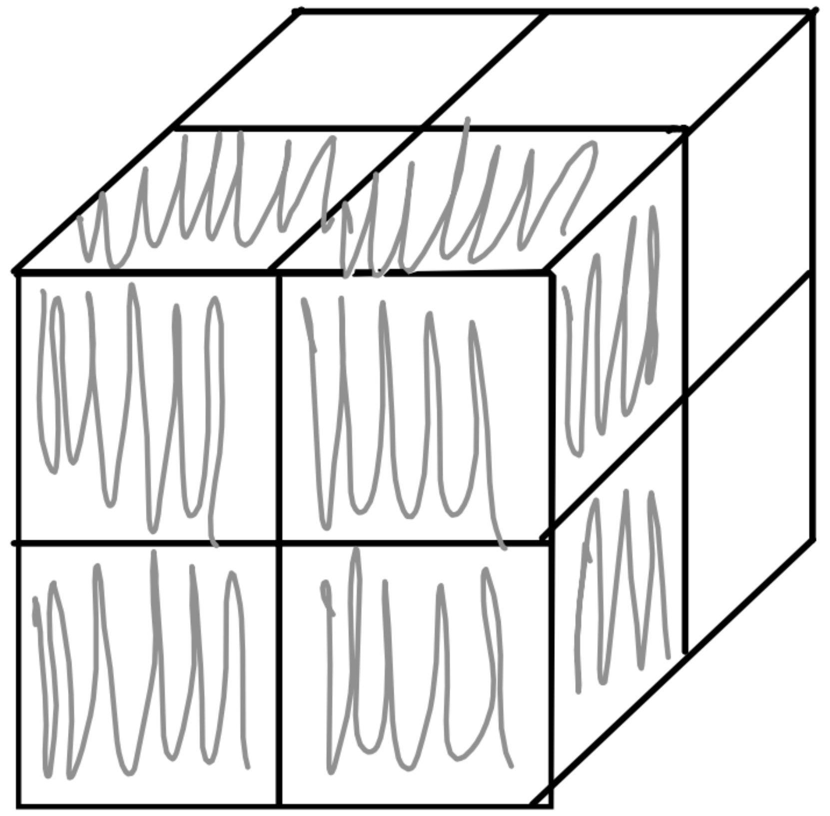


(2,1)

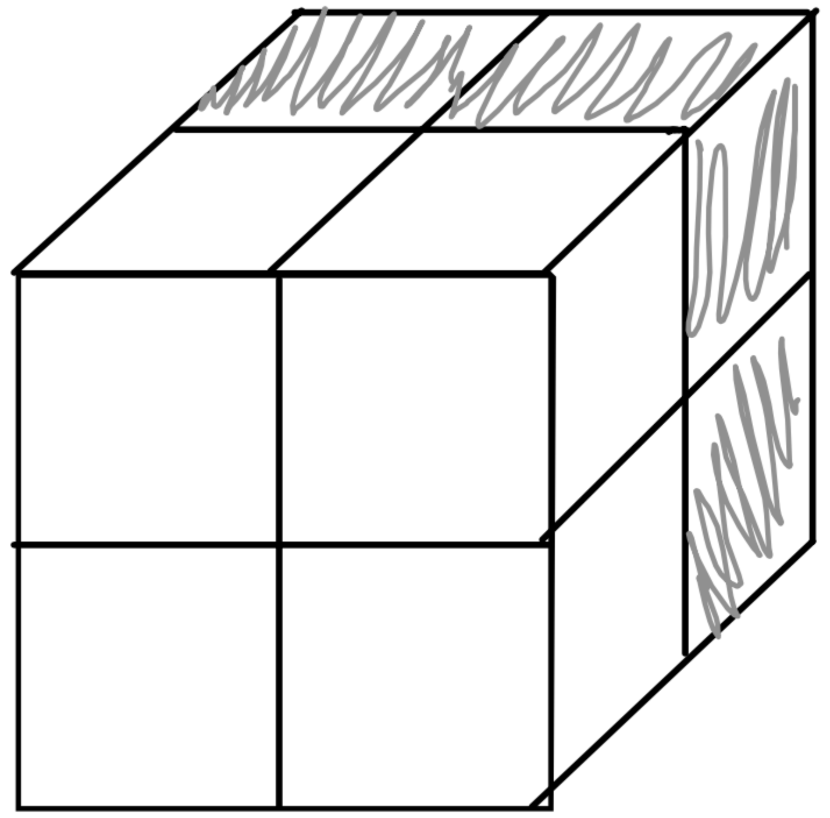


(2,2)

MPC 3



(3,1)



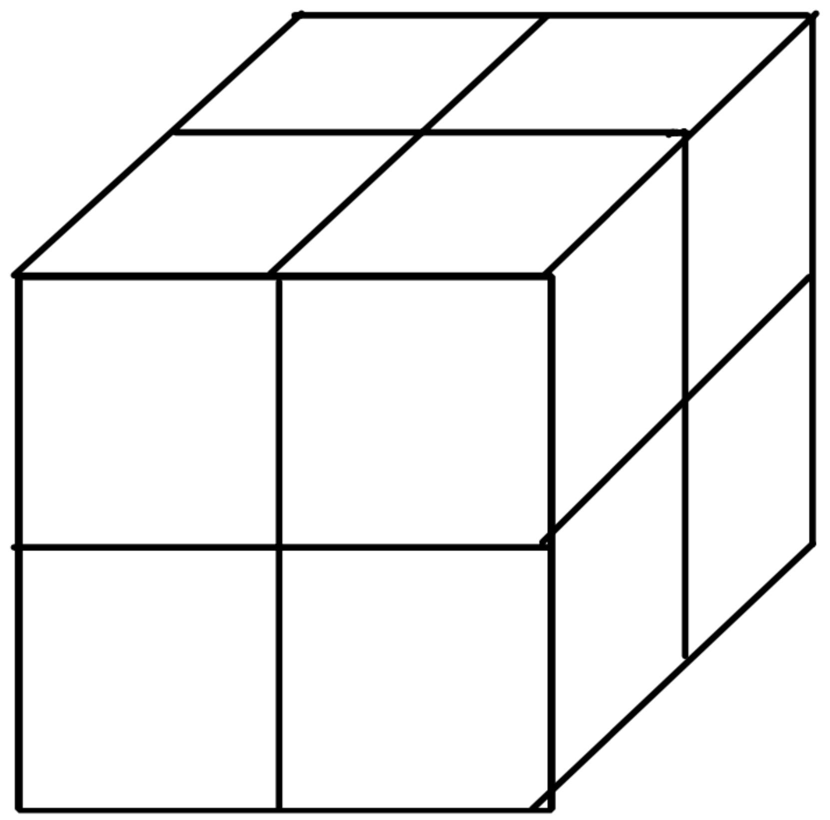
(3,2)

Hypercube

Take shares whose k-th coordinate is j.

Going to higher dimensions

Example. $D = 4$



Hypercube

Take shares whose k-th coordinate is j.

Going to higher dimensions

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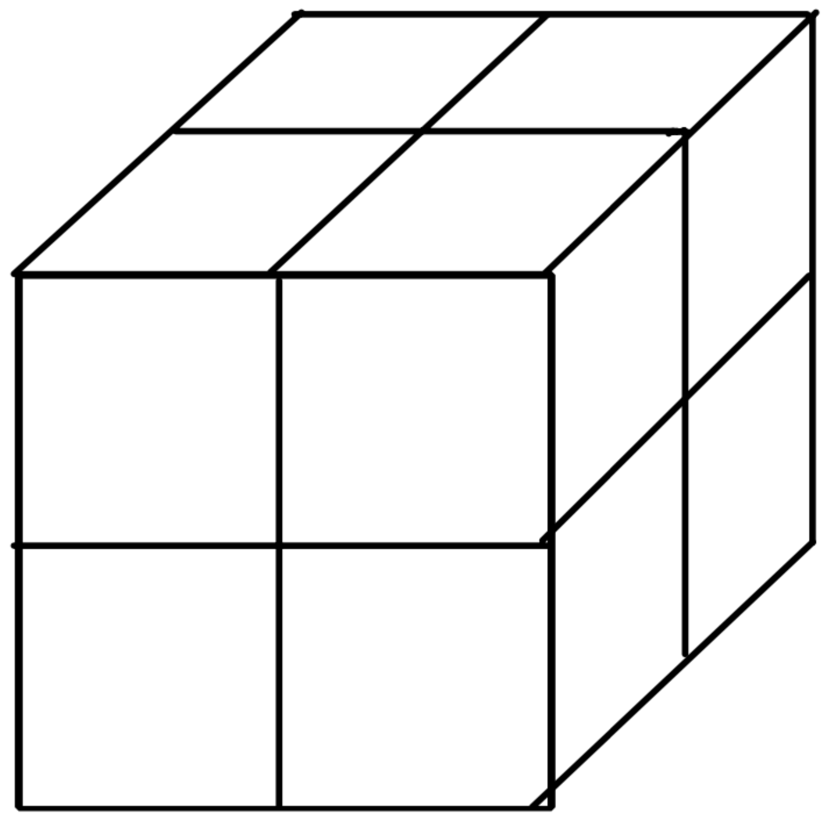
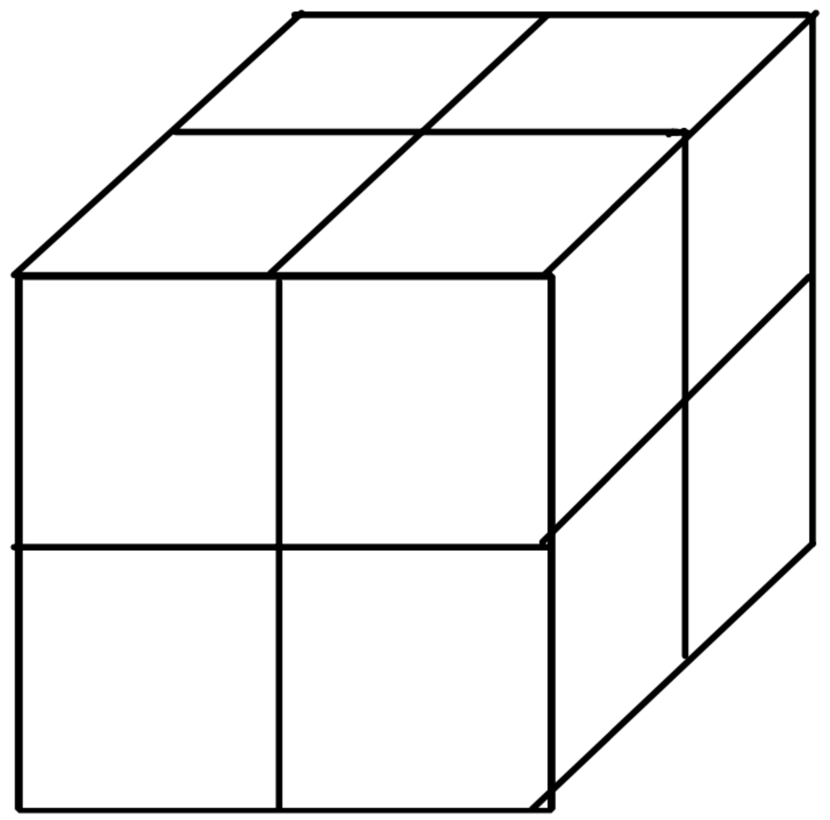


Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 4$



Hypercube

Take shares whose k-th coordinate is j.

Going to higher dimensions

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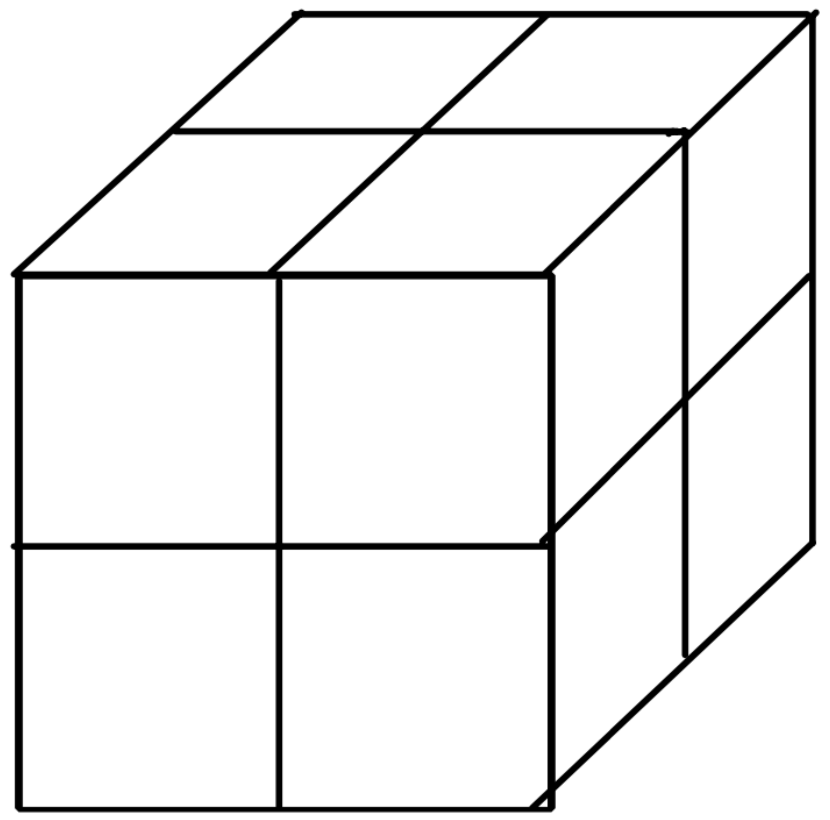

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Hypercube

Going to higher dimensions

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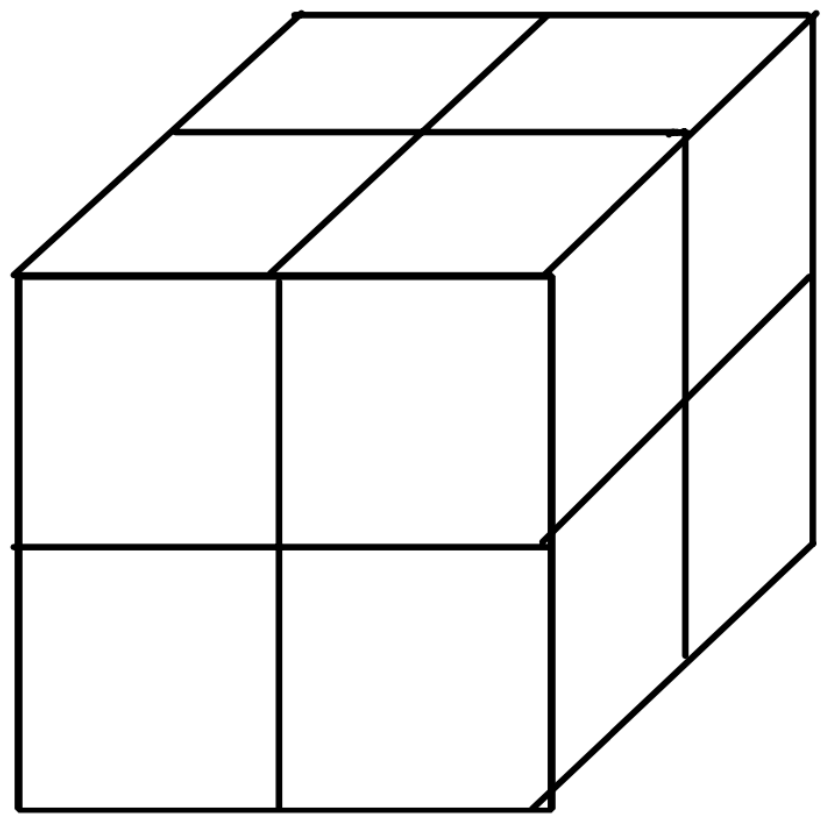

Hypercube

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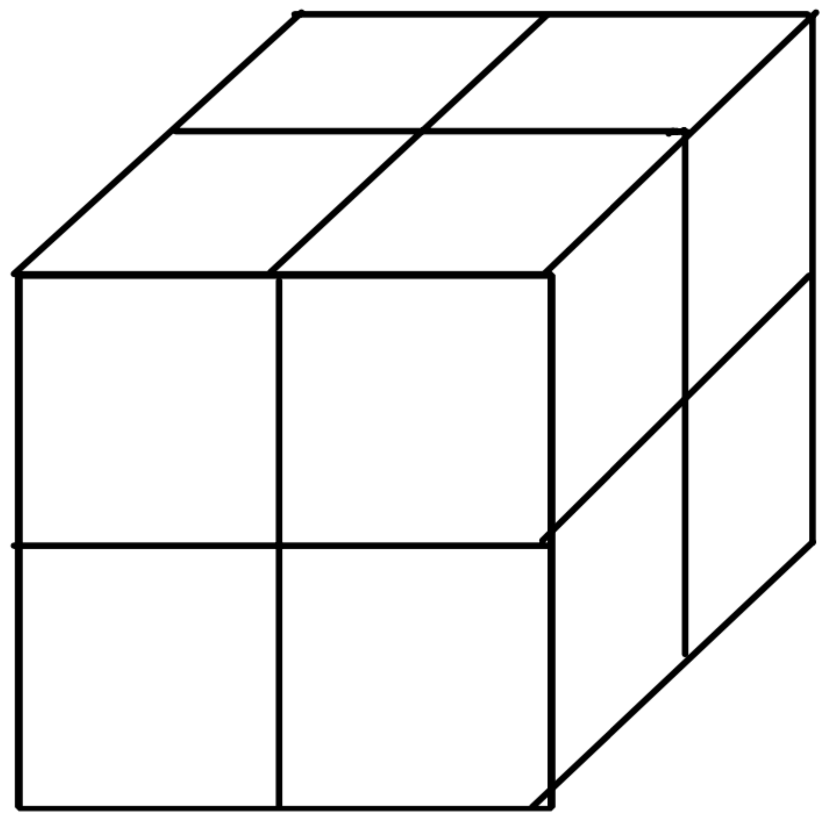

Hypercube

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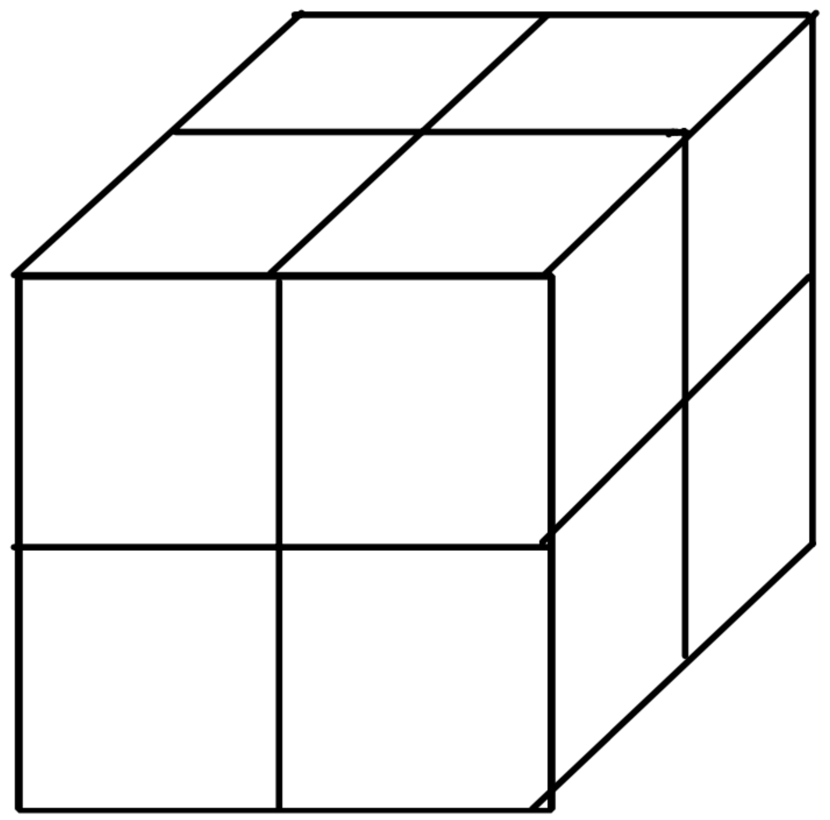

Hypercube

Take shares whose k-th coordinate is j.

Going to higher dimensions

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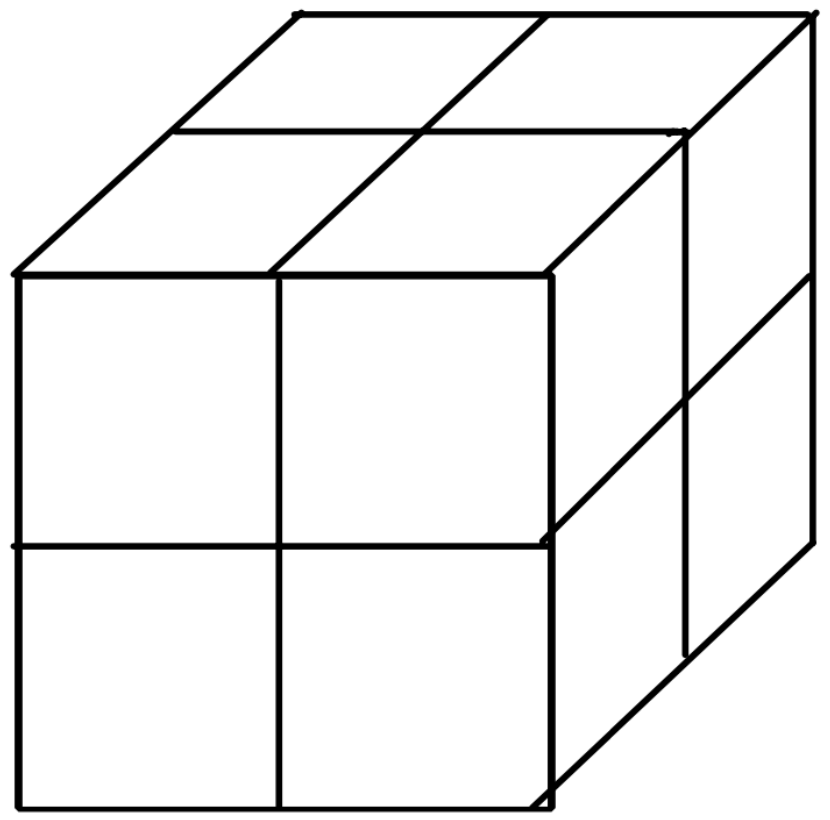


Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 4$

MPC 1

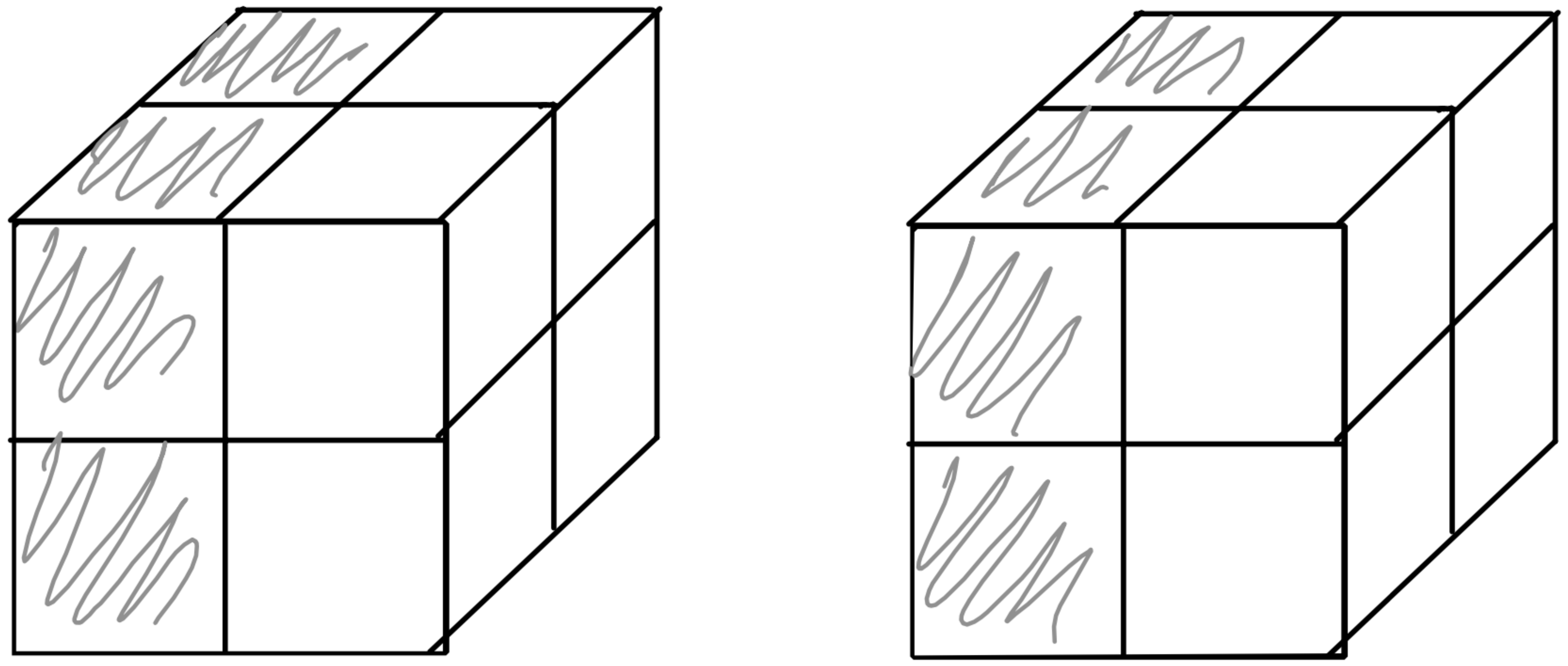
Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 4$

MPC 1



(1,1)

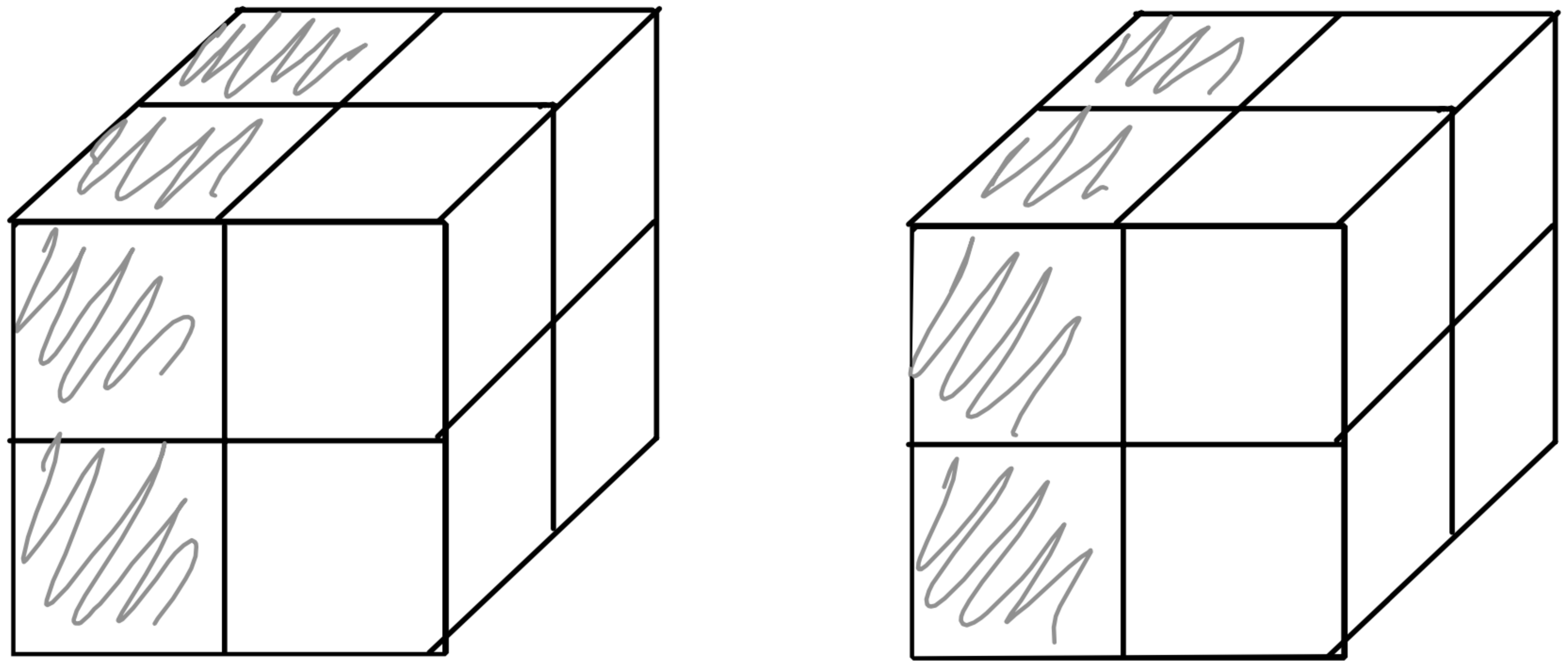
Hypercube

Take shares whose k-th coordinate is j.

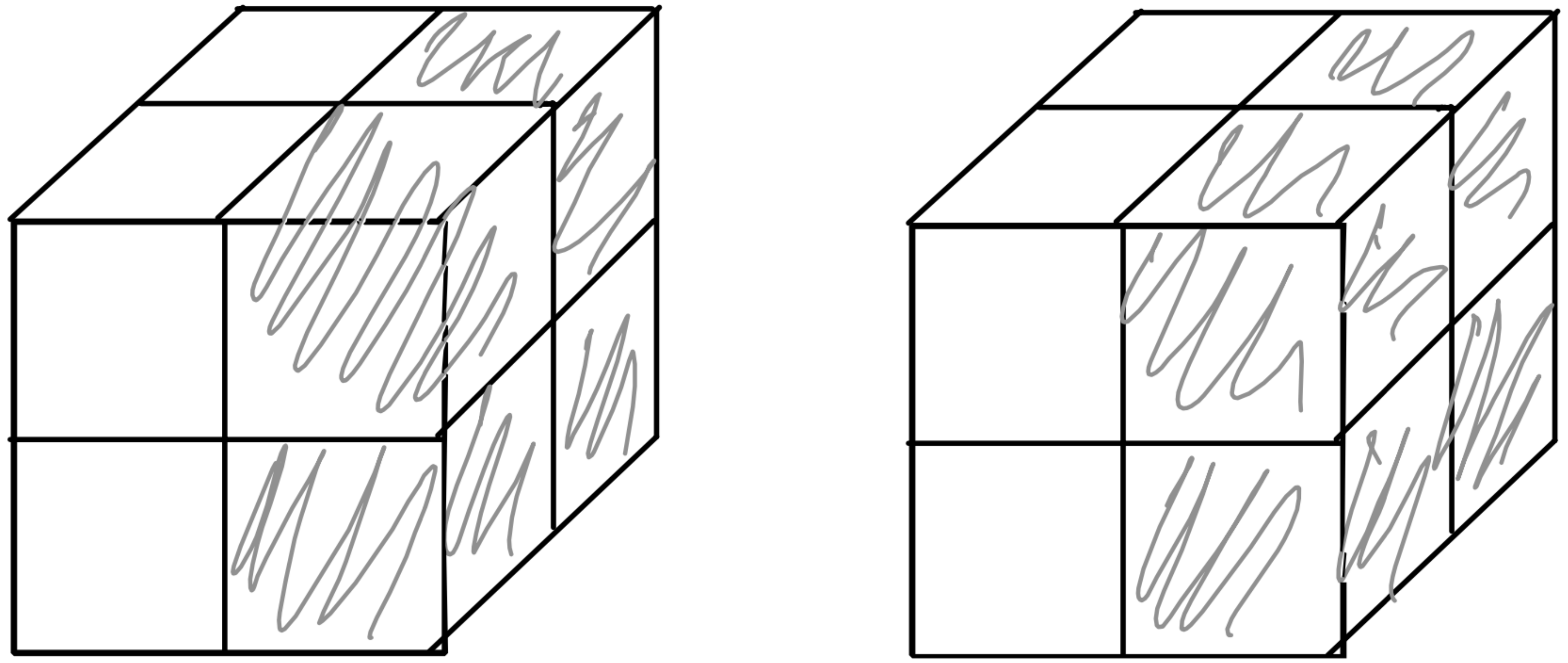
Going to higher dimensions

Example. $D = 4$

MPC 1



(1,1)



(1,2)

Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 4$

MPC 2

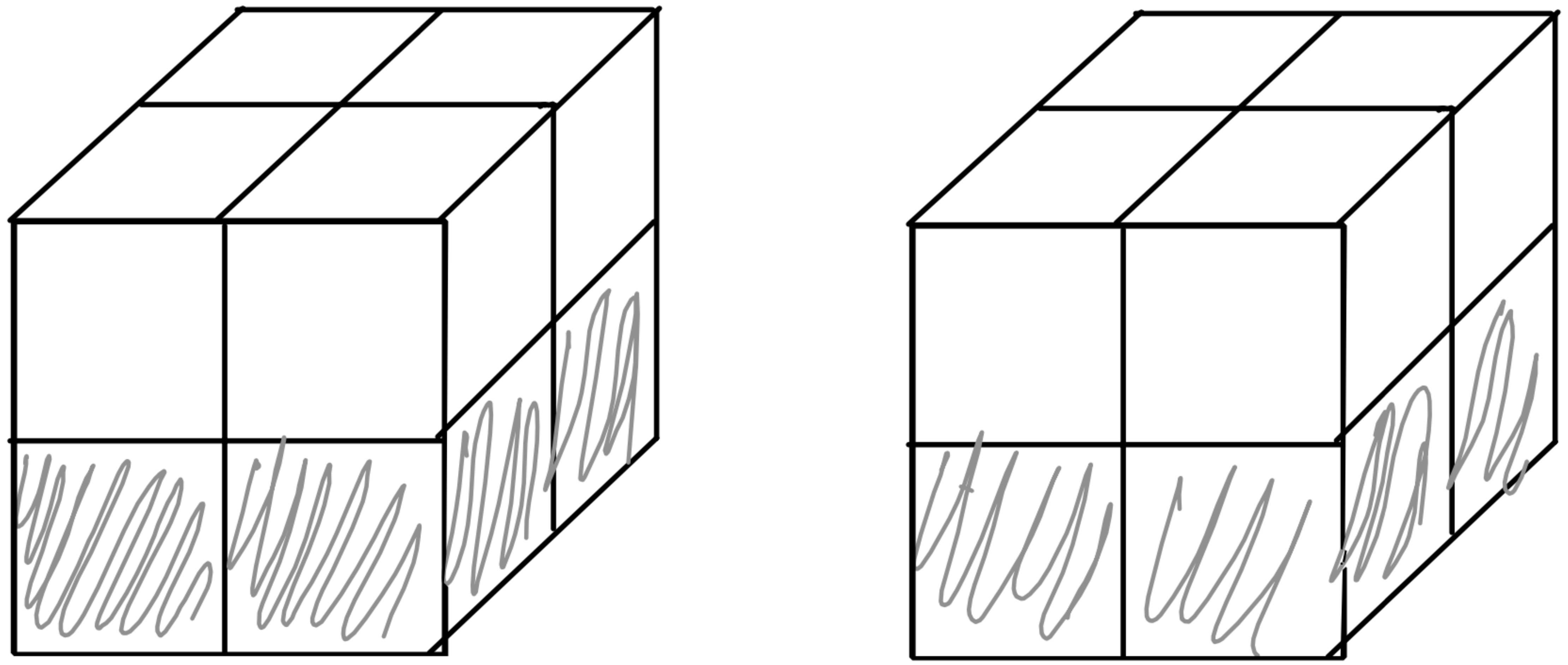
Hypercube

Take shares whose k-th coordinate is j.

Going to higher dimensions

Example. $D = 4$

MPC 2



(2,1)

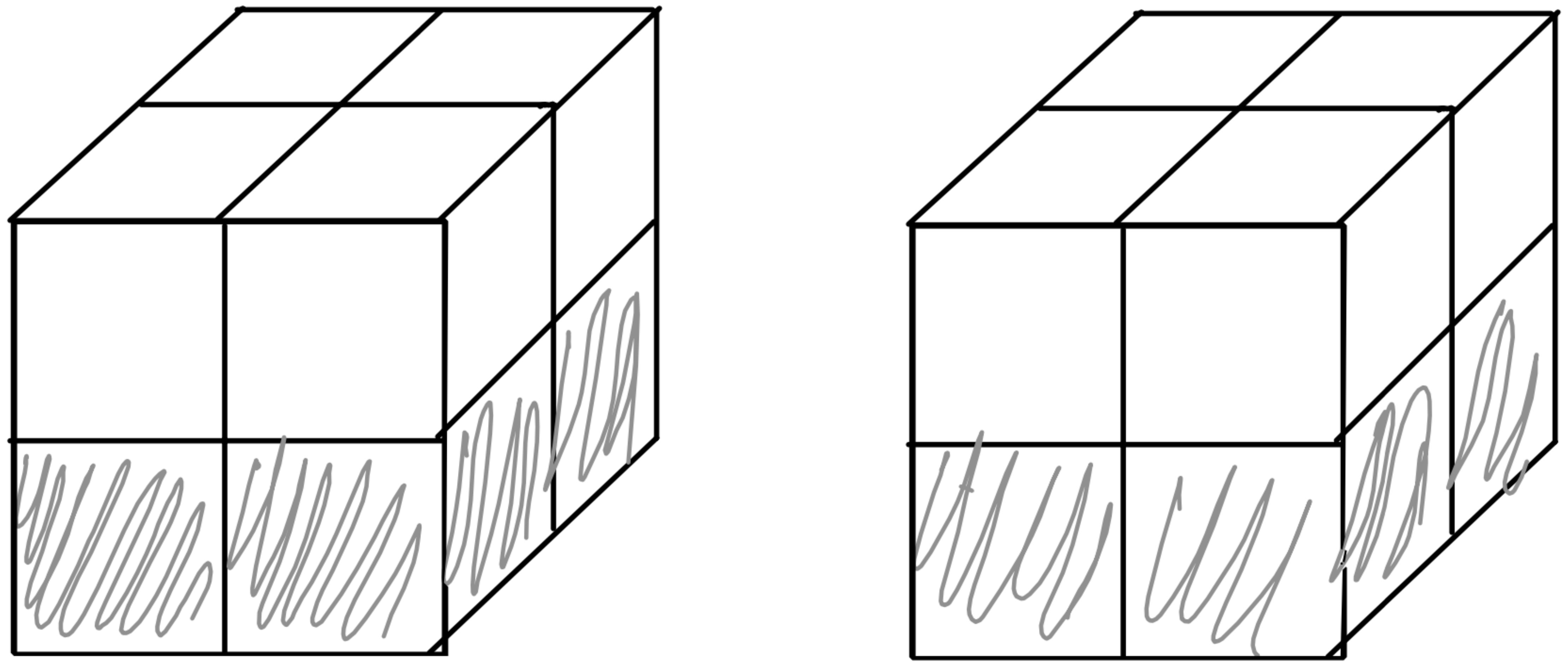
Hypercube

Take shares whose k-th coordinate is j.

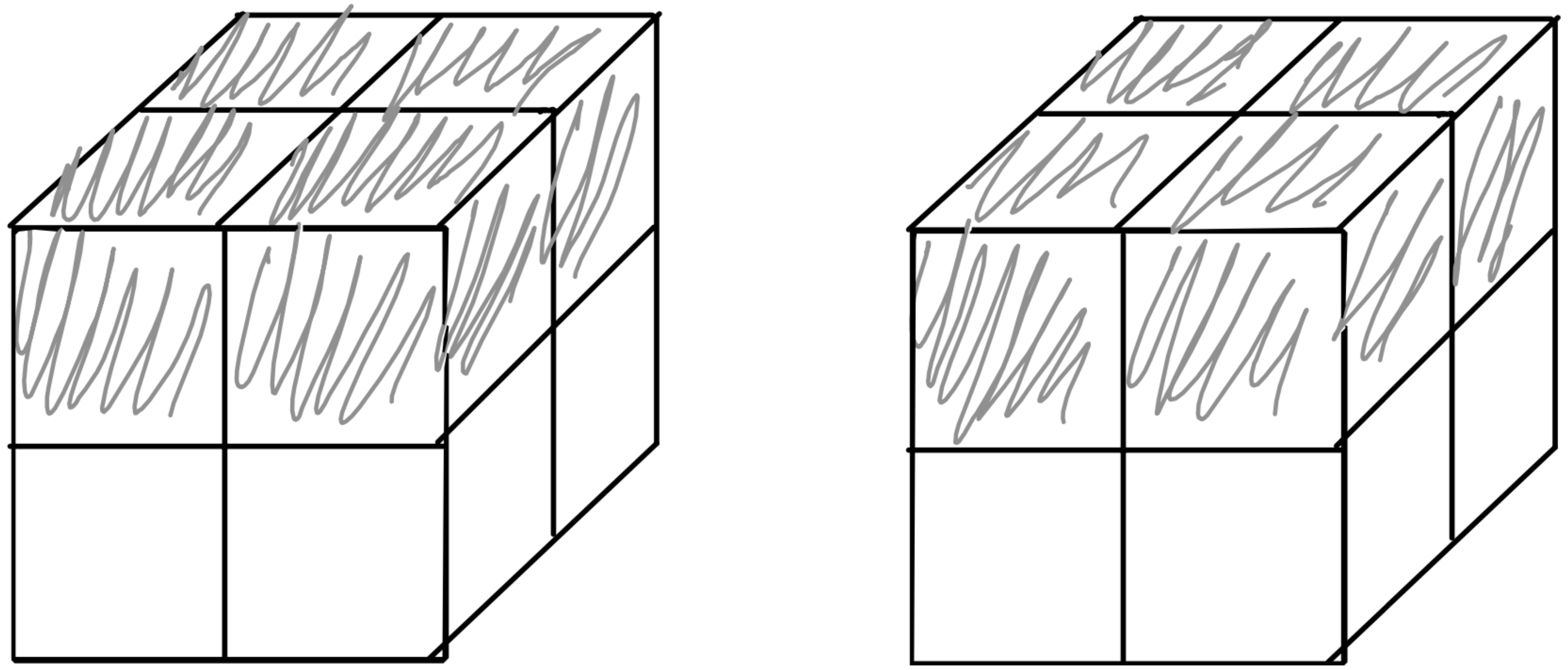
Going to higher dimensions

Example. $D = 4$

MPC 2



(2,1)



(2,2)

Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 4$

MPC 3

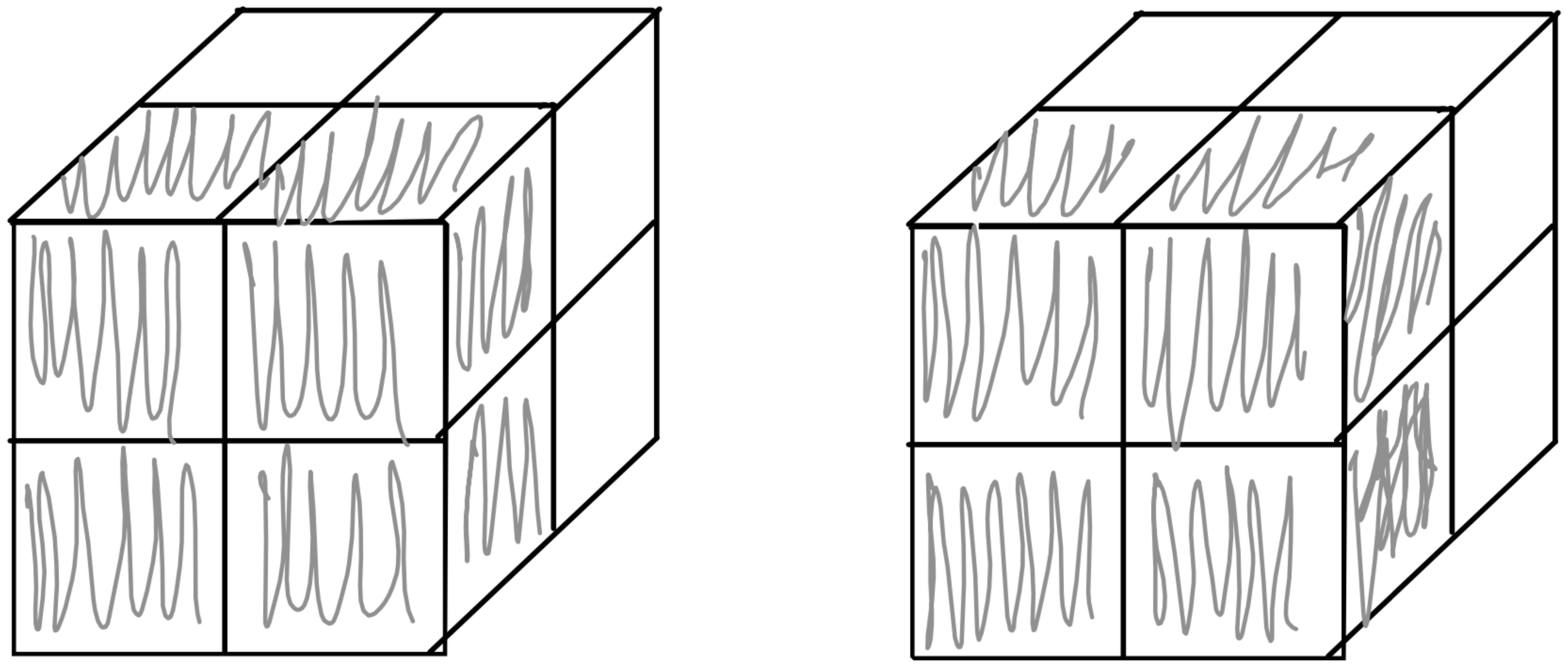
Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 4$

MPC 3



(3,1)

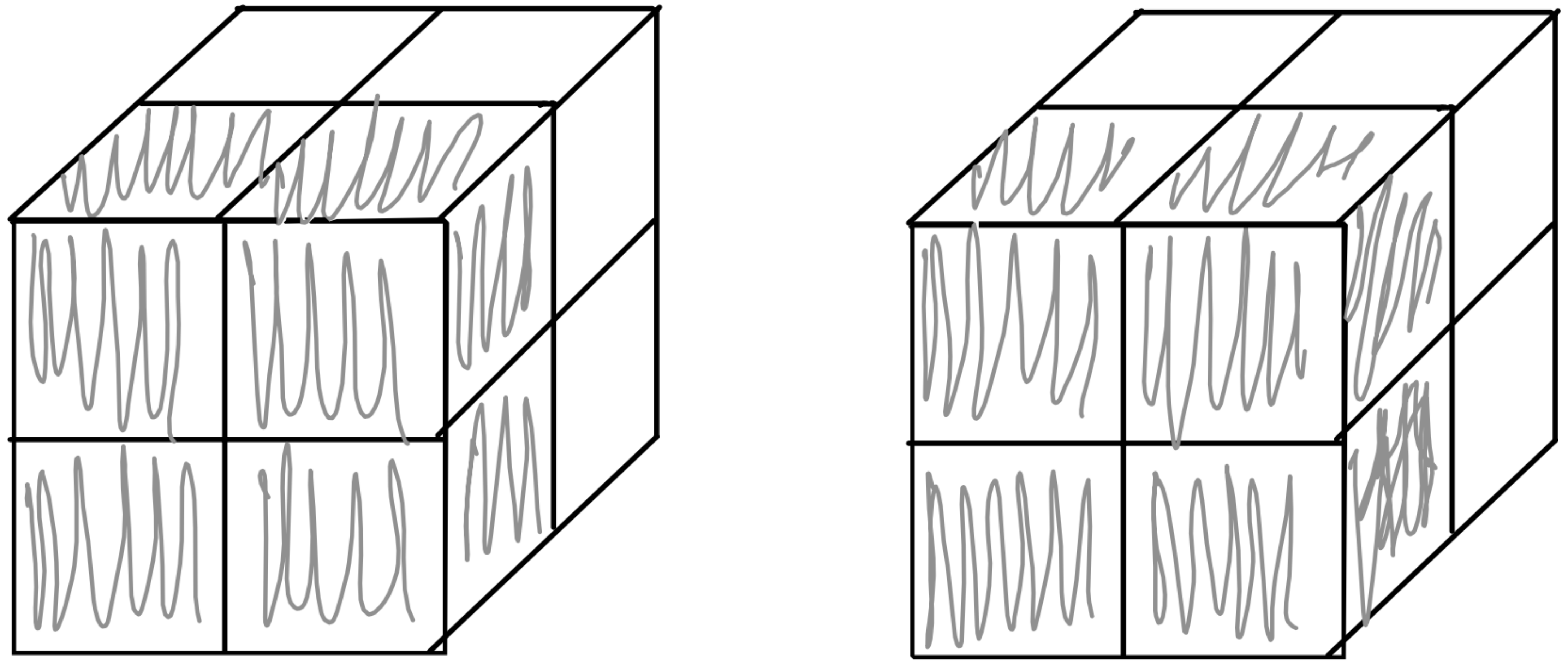
Take shares whose k-th coordinate is j.

Hypercube

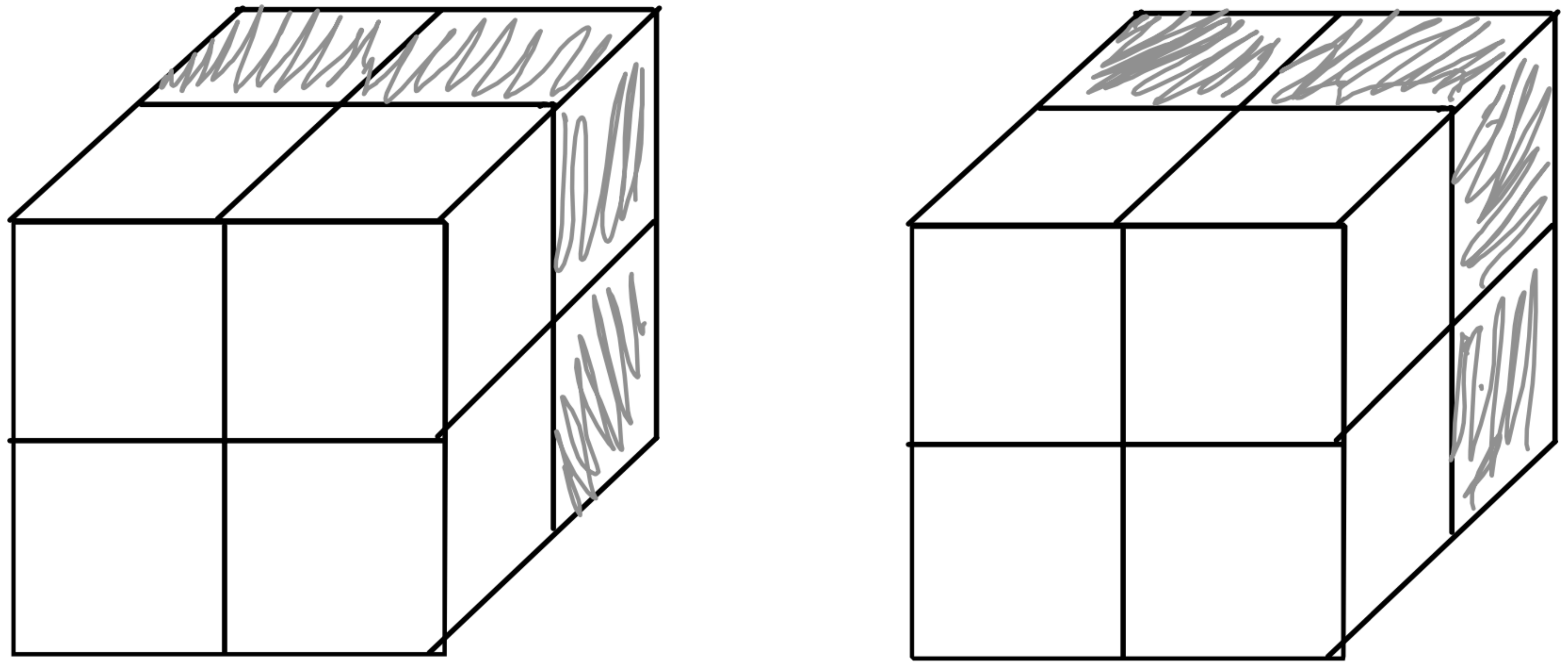
Going to higher dimensions

Example. $D = 4$

MPC 3



(3,1)



(3,2)

Take shares whose k-th coordinate is j.

Hypercube

Going to higher dimensions

Example. $D = 4$

MPC 4

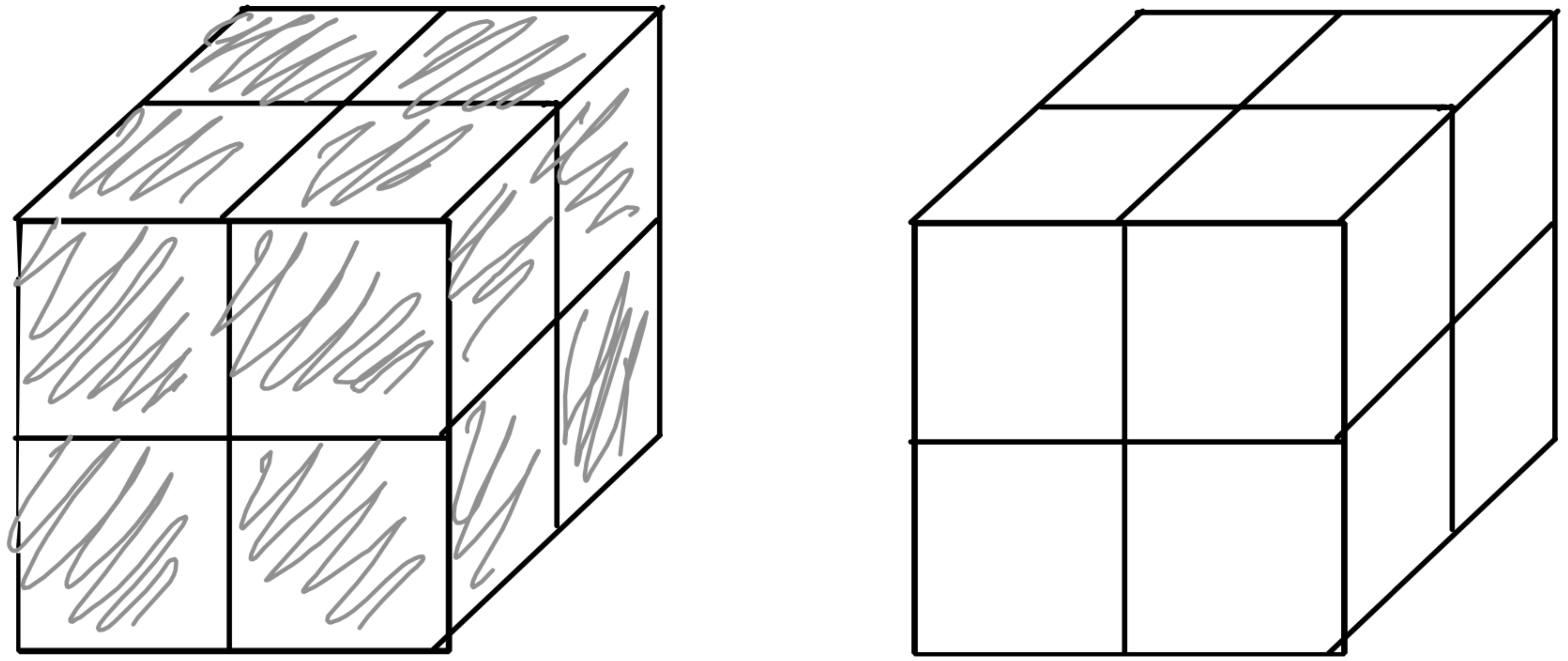
Hypercube

Take shares whose k-th coordinate is j.

Going to higher dimensions

Example. $D = 4$

MPC 4



(4,1)

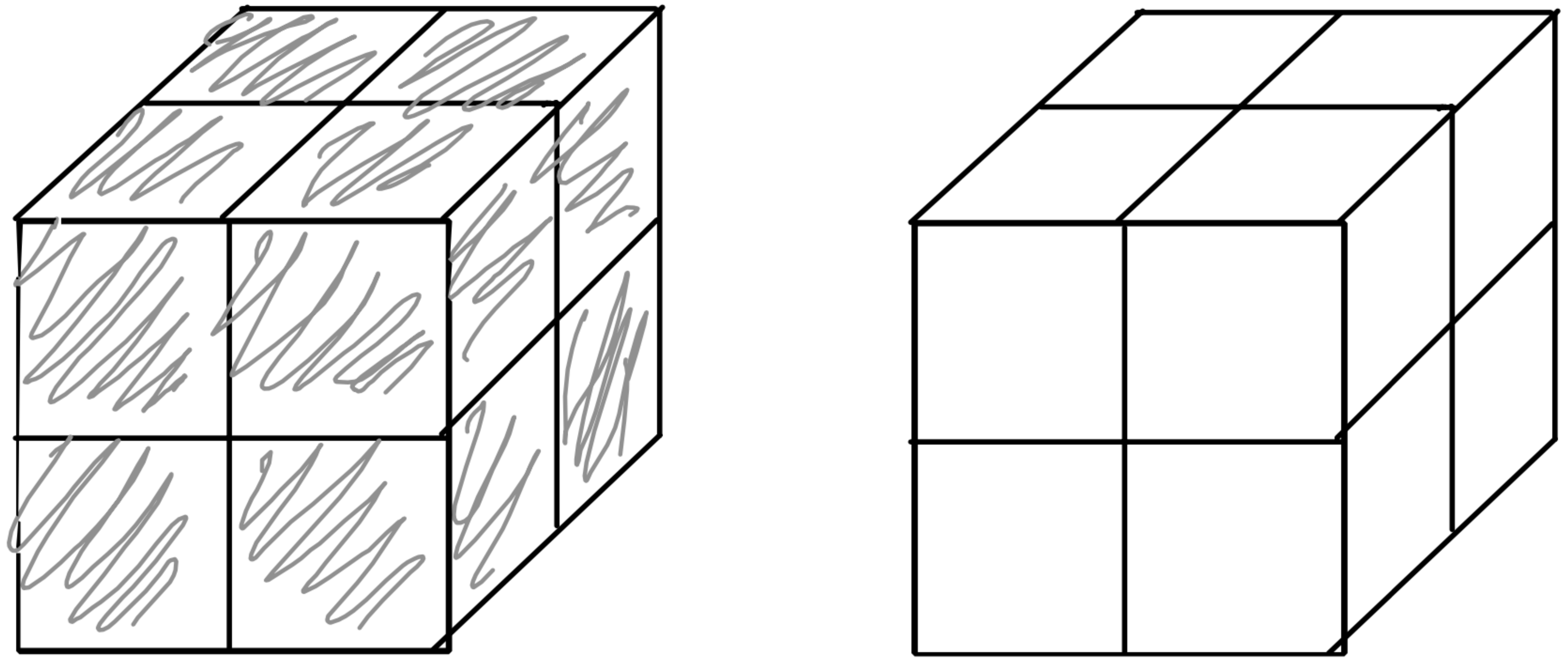
Take shares whose k-th coordinate is j.

Hypercube

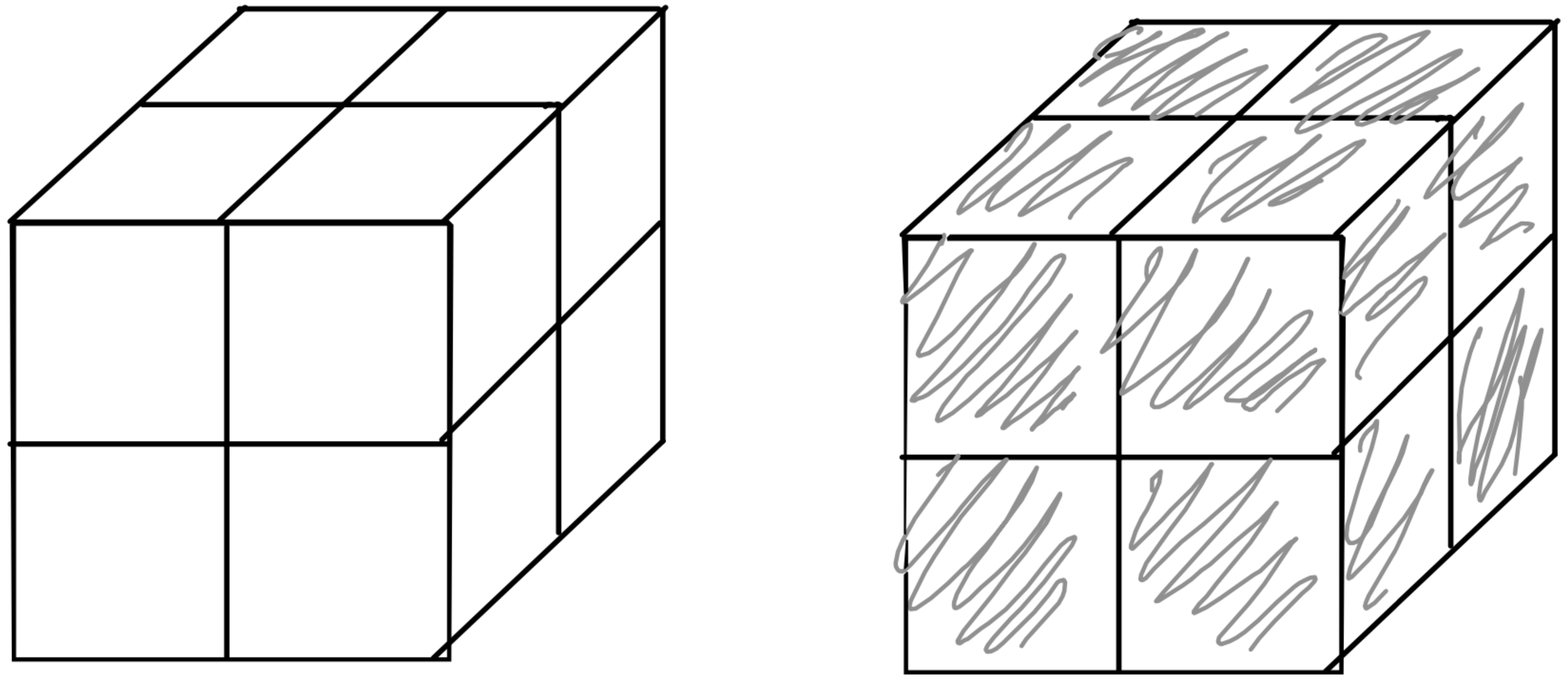
Going to higher dimensions

Example. $D = 4$

MPC 4



(4,1)



(4,2)

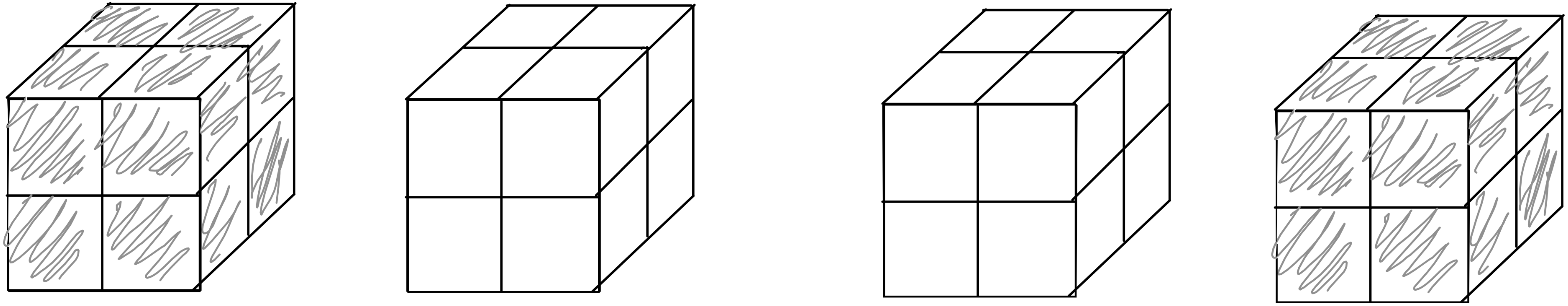
Hypercube

Take shares whose k-th coordinate is j.

Going to higher dimensions

Example. $D = 4$

<u>MPC 4</u>



(4,1)

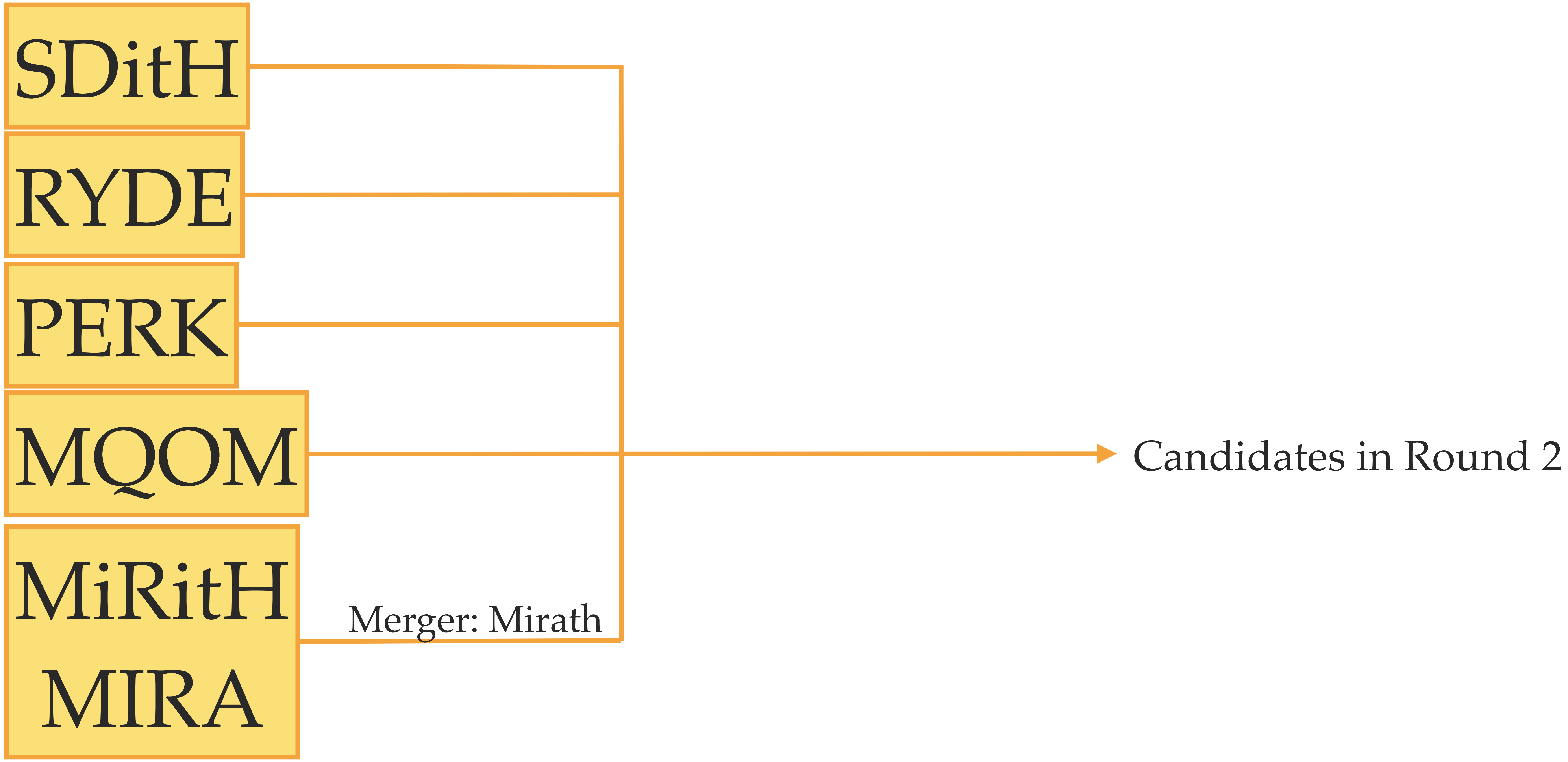
(4,2)

4 MPC instance of 2 main parties.

MPCitH in the NIST competition

SDitH	$\longrightarrow$ Syndrome decoding problem in the Hamming metric
RYDE	$\longrightarrow$ Syndrome decoding problem in the rank metric
PERK	$\longrightarrow$ Permuted kernel problem
MQOM	$\longrightarrow$ MQ problem
MiRitH	$\longrightarrow$ MinRank problem
MIRA	$\longrightarrow$ MinRank problem
Biscuit	$\longrightarrow$ MQ problem (with additional structure)
AIMer	$\longrightarrow$ MPC-friendly symmetric primitive

MPCitH in the NIST competition



Biscuit

AIMer